Essays in Macroeconomics

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Dedication

To my parents for making this possible.

Abstract

This dissertation consists of two essays. In the first essay, Enoch Hill and I develop a dynamic general equilibrium model in which an increase in the importance of firm-specific human capital is able to account for two key changes in business cycle patterns in the U.S. since mid-1980s: jobless recoveries and the reversal in the cyclicality of labor productivity. Additionally, we present empirical support that the importance of firm-specific human capital has indeed increased in importance for recent recessions.

In the second essay, Zhifeng Cai and I develop a macroeconomic model of financial frictions in order to account for the investment and cash holding behavior of self-financing corporations during the Great Recession. Unlike standard models of financial frictions which impose collateral or borrowing constraints on firms, the financial frictions in our model work through the liquidity channel. In our model, corporate investment is subject to liquidity shocks. Bank credit line and liquid assets are substitutes for financing liquidity shocks. In our model, a tightening of the bank credit line forces firms to hold more liquid assets, increasing the effective cost of capital expenditure hence reducing corporate investment.

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Chapter 1

Cautious Hiring

1.1 Introduction

It took nearly eight years following the 2008 recession for employment to reach its prerecession level. This slow rate of recovery in employment is a phenomenon shared by the two previous recessions (1990 and 2001). However, it contrasts sharply with recessions prior to the mid-1980s, in which recoveries in employment are relatively fast.[1] In fact, for recessions between World War II and the mid-1980s, employment recovered to pre-recession levels in an average of just over two years and always within four. This emergent pattern in employment is directly related to a conversation commonly referred to by policymakers, journalists, as well as in the economic literature, as *jobless recovery*.[2] Jobless recoveries refer to the periods following recessions in which rebounds in aggregate output are accompanied by much slower recoveries in employment.[3]

Closely related to jobless recovery is the cyclicality of average labor productivity. When the recovery in employment lags the recovery in GDP, labor productivity rises. In fact, during the three most recent recessions, average labor productivity has been countercyclical, as opposed to being procyclical as in earlier recessions.[4]

[1] See Gali et al. (2012).

[2] Examples include Gordon (1993), Bernanke (2003), Gali et al. (2012), and Jaimovich and Siu (2012).

[3] This definition is taken from Jaimovich and Siu (2012). A reconciliation of this defintion with Gali et al. (2012) is included in section 1.2.

[4] This has previously been documented by McGrattan and Prescott (2012), Gali and van Rens (2010), and Berger (2012), among others.

We propose a novel mechanism that accounts for the slower rate of recovery in employment and also reverses the cyclicality of labor productivity. Our mechanism works through the increase in the importance of firm-specific human capital, namely, the relative productivity of senior to junior workers. At a very abstract level before providing a more detailed explanation, firm-specific human capital turns workers into a type of investment. Productivity shocks make this investment risky. As the importance of firm-specific human capital increases, the initial outlays of the investment become greater and the potential benefits become greater as well making it more important to avoid risk. One way to avoid that risk is to grow more slowly at the firm level (spreading outlays over time) which in aggregate leads to a slower employment recovery.

Further, workers are initially unproductive in new jobs. If hiring following a recession is immediate, the fraction of workers in new positions will spike, lowering labor productivity along with the decrease in output. If hiring is spread over a longer period, this procyclical force is reduced. As average firms size following a recession is reduced, decreasing returns to scale in labor provides an upward force on average labor productivity. In recessions featuring fast employment recovery, the downward force from the spike in new workers outweighs the upward force of smaller firm size. This can be reversed for recessions with slower recoveries in employment.

Our economy features a simple model of firm growth in the spirit of Solow (1956). However, in our model, growth occurs through the accumulation of human capital in the form of senior workers, rather than the accumulation of physical capital.

There are a few key ingredients of our model. First, our model focuses on the portion of skills that are firm-specific.[5] Second, production is subject to idiosyncratic shocks which generate a risk of bankruptcy. Third, the production technology exhibits decreasing returns to scale. We now provide a brief intuition of how our model ingredients collectively account for the slower employment recovery and reversal in the cyclicality of labor productivity.

Firm-specific human capital introduces a hold-up problem between firms and workers. This hold-up problem compresses wages relative to differences in marginal productivity and converts the hiring decision into a form of investment. Specifically, newly

[5] Mahone (2016) indicates that one-third to one-half of the training is not transferable across jobs. Our model focuses on firm-specific skills and abstracts from general skills. General skills are fully compensated in the wage and have no effect on hiring decisions.

hired workers are overpaid and senior workers are underpaid relative to their contribution to output.

Intuitively, a newly hired worker gradually acquires firm-specific skills through learning-by-doing and transitions into a senior worker. A senior worker is more productive but has the same outside option as the newly hired worker. This is because with firm-specific skills, separation of senior workers from their firm results in a loss of all firm specific skills. If the worker were to separate they would have to start over as a new worker at another firm. Due to this low outside option, wage bargaining implies that firms are able to pay their senior workers a wage that is below marginal productivity. In other words, firms are able to reap positive surplus off their senior workers. In anticipation that new workers will transition into senior workers where positive surplus can be reaped, competition in the labor market will push the wage for new workers above their marginal productivity. Therefore, firm-specific skills convert labor hiring into a form of investment: a loss is incurred when a worker is newly hired in the form of overpayment, and this initial loss is recovered when the worker acquires specific skills.

This investment perspective of hiring, combined with an idiosyncratic risk of bankruptcy, leads to cautious hiring. A cost is incurred in the period junior workers are hired, and a profit is made later when these junior workers become senior. Consequently, the stock of senior workers is valuable to the firm. Hiring new workers lowers average profits of a firm in the period of hiring and increases the risk of bankruptcy. Bankruptcy is costly to a firm since it causes a separation from their stock of senior workers. Firms trade off a higher rate of growth with a higher risk of bankruptcy. As a result, firms hire cautiously and grow slowly.

As firm-specific skills increase in importance, the productivity gap between senior and junior workers widens, which leads to larger upfront costs as well as larger subsequent returns. This raises the stakes of hiring and inherently makes it more risky in nature. As a result, firms hire more cautious and grows more slowly. Following a recession, a larger than normal fraction of firms exit which leads to a drop in aggregate employment. Recovery is driven by new firms starting from size zero and gradually growing to the mature size. Therefore, slower firm growth translates into a slower rate of employment recovery.

The key parameter in our model is the relative productivity of senior to junior workers. As this ratio increases, firms are incurring more costs on each junior worker and earning more profits off each senior worker. This makes hiring more risky: each junior worker hired reduces the probability of firm survival, and the stock of senior workers is more painful to lose if the firm fails to survive. Therefore, increasing the relative productivity of senior to junior workers endogenously slows firm hiring. Our model focuses on the recovery following a recession. In the context of our model, a recession results in a larger than normal fraction of firms to exit. Recovery is driven by new firms that start from size zero and grow to the mature size. Slower firm hiring translates into a slower rate of employment recovery following a recession.

Next, we discuss how our mechanism is able to reverse the cyclicality of average labor productivity. When the economy is recovering from a recession, there are two opposing forces on labor productivity. The first is related to the composition of senior to junior workers employed in the economy. Following an unexpected shock, a large number of senior workers are separated from firms. Firms gradually hire back junior workers. Therefore, in the ensuing recovery, the proportion of junior workers temporarily increases. This exerts downward pressure on average labor productivity, since junior workers are less productive than senior workers. The more quickly firms hire during recovery, the stronger this composition force is.

The opposing force on labor productivity works through the decreasing returns to scale production technology. A recession causes a fraction of firms to exit. These firms start over from size zero, engage in cautious hiring, and are smaller in size than an average firm in the stationary equilibrium. Due to the decreasing returns to scale production technology, a smaller average firm size drives up average labor productivity.

When firm-specific human capital is less important, firms grow fast. This large influx of newly hired workers strengthens the composition effect, resulting in a procyclical pattern for average labor productivity. When firm-specific human capital becomes more important, firms hire more slowly. Slower hiring spreads the influx of new workers across time, and causes firms to spend a longer time operating with fewer workers. Therefore, the composition effect is weakened and the decreasing return effect is strengthened. As a result, a countercyclical pattern of average labor productivity emerges.

Of crucial importance to our model is the importance of firm-specific human capital, and how it evolves over time. We estimate returns to tenure from the PSID, and use it to infer the importance of firm-specific human capital.[6] We find that wage returns to average tenure have increased from 4.84% for the period prior to 1982 to 13.02% for the period after.

We calibrate a quantitative version of the model and show that the estimated increase in the importance of firm-specific human capital can account for a quarter of the slower employment recovery, and also reverses the cyclicality of labor productivity.

1.2 Related Literature

In this section, we briefly explain how our work ties into the existing empirical and theoretical literature regarding slow labor recoveries and emergent acyclical labor productivity patterns.

A number of papers including Gali et al. (2012), Gordon (1993), Groshen and Potter (2003), and Bernanke (2003) have documented the slower rate of recovery in labor through recent business cycles. Our model explains this change through an increase in the importance of firm-specific human capital since the mid-1980s.

Our theoretical explanation for the slow labor recovery overlaps with several others via the introduction of a mechanism which separates the marginal productivity of labor from the wage. For example, Hall (2017) and Midrigan et al. (2014) emphasize the role of discount rates. In their models, hiring a new worker incurs an upfront vacancy posting cost which is earned back from future differences between productivity and wage. If somehow the effective discount rate of firms falls (that is, future profits are valued less today) during the recession (for example, due to a tightening of borrowing constraints as in Midrigan et al. (2014)), hiring new workers becomes less profitable. Similar to their models, our model also features an upfront cost of hiring new workers. The difference is that in our model, this cost is due to the lower productivity of newly hired workers relative to their wage. In addition, our mechanism for generating the slower recoveries in recent recessions is an increase in the gap between the productivity levels of new and existing workers, instead of a reduction in the effective discount rate. An added benefit of our model is its ability to explain the emergent pattern in labor

[6] We follow methods used by both Altonji and Williams (2005) and Topel (1991) to estimate the returns to seniority over time.

productivity over the business cycle.

Our mechanism does not fully account for the slower recovery in employment observed in the data. Complementary explanations include worker reallocation (Groshen and Potter (2003)), the greater use of just-in-time employment practices (Schreft et al. (2005)), permanently destroyed employment at large firms (Luttmer (2012)), increased access to credit card borrowing (Herkenhoff et al. (2016)), extension of countercyclical unemployment benefits (Rabinovich and Mitman (2012)), rigid real wages due to either lack of inflation or efficiency wages (Bils et al. (2016), Calvo et al. (2014), Shimer (2012)), changes in female labor force participation and gender-biased technological change (Sahin and Albanesi (2013)).

In addition to explaining the slow recovery, our work is related to the literature on the cyclicality of average labor productivity. Prior to the mid-1980s, labor productivity was procyclical. Explanations for this procyclical pattern include labor hoarding (for example, Oi (1962) and Fay and Medoff (1985)) and procyclical technology shocks in standard RBC models (for example, Kydland and Prescott (1982)).

Documentation and explanations for the new pattern in average labor productivity starting from the mid-1980s include Mulligan (2011), McGrattan and Prescott (2014) and McGrattan and Prescott (2014). To explain the new pattern in labor productivity, McGrattan and Prescott (2014) incorporates intangible capital into a standard RBC model, Arellano et al. (2016) shows that increased uncertainty at the firm level slows down firm hiring without dragging down labor productivity, Gali and van Rens (2010) demonstrates lower labor adjustment cost results in less labor hoarding hence less procyclical average labor productivity, and Barnichon (2010) attributes declined cyclicality of labor productivity to declined aggregate demand shocks.[7]

[7] As discussion of new patterns in labor productivity are core to our model, a brief aside to consider the findings of Gali et al. (2012) is warranted. Gali et al. (2012) prefer to describe the emergent patterns following recent recessions as *slow recoveries* rather than *jobless recoveries* since recoveries of GDP (and several other series) have also slowed following recent recessions. We agree that this finding warrants additional recognition in the literature by corroborating this finding in the data and replicating the slower recovery in GDP through our model. However, we also argue that the relationship between GDP and labor productivity has fundamentally shifted. Gali et al. (2012) rely on aggregate statistics observed at four and eight quarters following each recession to establish changes in recovery patterns. Our observations on average labor productivity make use of the time series as well as the correlation statistic. Our findings reveal that the pattern in average labor productivity following a recession is not monotonic which implies that discrete observations at various points could mask underlying changes. Observations of the correlation between GDP and labor productivity, which can be observed in Figure

Most related to our paper are those which attempt to provide explanations for both the jobless recoveries and the change in the cyclicality of labor productivity. Sims and Pries (2011) suggest that the nature of aggregate shocks has grown more asymmetric across sectors. This leads to a reallocation of labor across sectors and a slow recovery. Our mechanism can also be motivated through sectoral reallocation in that workers reallocate from routine sectors into non-routine sectors. Our paper is also related to Berger (2012) which attributes changes in average labor productivity to firm restructuring and the removal of less productive employees during recoveries. In both papers, hiring workers is initially costly but profitable later on. In Berger (2012), only certain new employees are good matches. As a result, hiring new employees is costly and the existing stock of workers is valuable. In our model, senior employees have acquired firm-specific skills which are not fully compensated in the wage. These workers represent a real value to the firm.

Our mechanism relies on the consideration of a firm's hiring decision as a form of investment. In this sense, our paper builds on the seminal work of Becker (1964). In this work, the costs and returns of firm-specific training are largely attributed to firms and there is a difference between the marginal product of labor and the wage. Additional empirical work on this topic includes Frazis and Loewenstein (2006), and Isen (2012). Our mechanism is also at work in Hudomiet (2015), where adaptation costs are incurred due to the lower productivity of newly hired workers. This leads to fewer jobs and a longer unemployment duration. However, our application differs from the work of Hudomiet (2015) in its exploration of how differences in adaptation costs across various occupations generate higher unemployment rates for less educated workers.

Firm-specific human capital is of critical importance to our mechanism. We use the returns to seniority as a proxy for firm-specific skills and estimate it following the methodology of Altonji and Shakotko (1987), Altonji and Williams (2005), and Topel (1991). This is related to a large literature estimating the returns to seniority, including Buchinsky et al. (2010), Abraham and Farber (1987), Dustmann and Meghir (2005), and Neal (1995).

1.2, provide additional support that the relationship between GDP and average labor productivity has changed.

The rest of the paper is organized as follows. In Section 1.3, we document two main patterns which have changed over more recent recessions. Section 1.4 presents the dynamic model and defines the stationary equilibrium. In Section 1.5, we present our stationary equilibrium results. Section 1.6 calibrates our parameters to the data. In Section 1.7, we present the business cycle properties of our model and compare them to the data. Section 1.8 concludes.

1.3 Data

Employment recoveries following the mid-1980s have been significantly slower than those prior. Not only have employment recoveries become slower, they have also become slower *relative* to the corresponding recoveries in GDP. This can be observed through the cyclicality of average labor productivity. In this section, we formally present these emergent patterns with a particular focus on business cycles.

1.3.1 Employment Recovery

Employment is measured as total hours worked in the non-farm business sector using data from the BLS. To visually observe the slower recovery in employment, a comparison plot is provided in Figure 1.1. To assist in comparison, employment is normalized to zero at the pre-recession peak. Employment drops during each recession and gradually recovers afterwards. The blue solid line is the average path of employment for the three most recent recessions. The red dashed line is the average path for the three recessions immediately preceding the mid-1980s. This comparison highlights the slower recovery of the three more recent recessions.

To formally calculate the speed of employment recovery for different recessions, we compute the number of years it takes for employment to return to its pre-recession peak. To standardize comparison across business cycles, we measure the speed of employment recovery as the length of time between the NBER end date and the date at which employment returns to its pre-recession peak. Measuring recovery from the NBER end date avoids conflating the length of recession with the length of recovery. The result is displayed in Table 1.1.

For the recessions prior to the mid-1980s (1953, 1957, 1960, 1969, 1973 and 1981

Table 1.1: Employment Recovery Comparison

	Fast Recovery Recessions						
Recession Starting Year	1953	1957	1960	1969	1973	1981	Average
Recovery Length	2	1	1.25	2.75	4	1.25	2.04
	Slow Recovery Recessions						
Recession Starting Year	1990	2001	2007				
Recovery Length	> 6	6.5	> 6				

Source: Bureau of Labor Statistics, NBER recession dates, and Author's calculations

recessions[8]), employment fully recovers to pre-recession levels within four years. The average recovery length is 2.04 years. In contrast, even six years after each of the three most recent recessions (1990, 2001 and 2008 recessions), employment remains significantly below the pre-recession level.

Additional comparison in the speed of recovery in employment is included in Appendix A.1.

1.3.2 Average Labor Productivity

Not only have employment recoveries become slower for recent recessions, they have also become slower relative to the corresponding recoveries in output.[9] We define average labor productivity to be real output over total hours worked. If GDP recovers more slowly than employment following a recession, average labor productivity will stay below its stationary equilibrium level along the recovery path. Consequently, average labor productivity will be pro-cyclical. Alternatively, if GDP recovers faster than employment, average labor productivity will stay above its stationary equilibrium level along the recovery path and display a counter-cyclical pattern.

Figure 1.2 displays the 10-year centered moving-average correlation between labor productivity and GDP. Each point in the series is calculated as the correlation between GDP and average labor productivity for a window of 10 years centered at the displayed date. Prior to the mid-1980s, labor productivity was procyclical. However, following

[8] The recovery following the 1980 recession is interrupted by the 1981 recession, resulting in an incomplete recovery path.

[9] This fact has been previously documented. See Gali and van Rens (2010), Barnichon (2010), Berger (2012), and McGrattan and Prescott (2012) for examples.

the mid-1980s, labor productivity has been acyclical or even slightly counter cyclical.

To focus on the correlation between average labor productivity and GDP over the business cycle, we plot the comovement of the two data series from each of the six most recent recessions. In the three recessions immediately prior to the mid-1980s (which we will label the "Fast Recovery" recessions), average labor productivity tracks GDP closely: dropping at or just before the onset of the recession and recovering fairly quickly following the recession. However, in the three most recent recessions (the "Slow Recovery" recessions), the comovement between average labor productivity and GDP becomes weaker. Average labor productivity remains fairly flat leading into the recession, increases rapidly beginning mid-recession, and falls a few years after the recession. These features can be observed in Figures 1.3 and 1.4.

1.4 Environment

Our economy features a simple model of firm growth. Unlike standard models where the growth of firms occurs through the accumulation of physical capital, in our model, growth occurs through the accumulation of human capital in the form of senior workers.

Time is discrete. There are three types of agents: a continuum of representative households, a continuum of heterogeneous firms, and financial intermediaries. The problems of households and financial intermediaries are simple. The primary focus is on firms.

1.4.1 Households

There is a unit measure of representative households. Households supply labor to firms and consume wage and dividend income. Formally, the household's problem is given by

$$\max_{\{c_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(C_t, L_t\right)$$
$$\text{s.t. } C_t \leq w_t L_t + D_t, \text{ for all } t$$

where w_t is the average wage income of the household and D_t is the dividends paid out by firms. The workforce is comprised of senior and junior workers who receive different wage rates. For simplicity, we assume that households are organized into families.

Each family has a representative share of senior and junior workers. Consequently, $w_t = w_t^J s_t^J + w_t^S s_t^S$, where w_t^J and w_t^S represent the wage rates of junior and senior workers. s_t^J and s_t^S represent the share of junior and senior workers employed in the economy. Similarly, each family also owns a representative share of the firms. This family structure removes idiosyncratic risk at the household level and allows us to focus on firms. (This risk-sharing arrangement is similar to Merz (1995), Andolfatto (1996), and Midrigan et al. (2014).)

In equilibrium, the resource constraint implies that consumption for each household is equal to the total output of the economy: $C_t = Y_t$.

1.4.2 Firms

There is a unit measure of firms owned by the households. They produce output according to the production function $y = zn^{\alpha}$. z is an idiosyncratic productivity shock that is i.i.d across firms with distribution function $F(z)$. n is the total effective units of labor hired by the firm.

At each firm, workers differ in terms of their firm-specific skills. During the initial period of employment at a firm, workers are junior without any firm-specific skills. Junior workers convert 1 unit of time into θ^J units of effective labor. At the end of each period of employment, junior workers acquire firm-specific skills and transition into senior with probability ζ. Senior workers convert 1 unit of time into θ^S units of effective labor. Senior workers stay senior unless they are separated from the firm. In the case of separation, senior workers lose all firm-specific skills and start over as a junior worker in the labor market. Since skills are firm-specific, firms are unable to directly hire senior workers. Instead, in order to acquire a senior worker, they must hire a junior worker who becomes senior in subsequent periods. A firm with n^S existing senior workers which decides to hire n^J junior workers has total effective labor units given by $n = \theta^J n^J + \theta^S n^S$.[10]

[10] Our model abstracts from general forms of human capital which can be transferred across firms. We assume that any forms of human capital which modify a worker's outside option are fully reflected in their wage.

One way our model implicitly includes general forms of human capital (albeit in a relatively rigid framework) is by assuming that a worker with h units of general human capital provides h units of effective labor of the category of their firm specific human capital. Firms choose how many units of effective junior labor to hire and pay w^J for each unit. A junior worker with h units of general human

In the spirit of Calvo (1983), modeling the transition from junior to senior worker as a probabilistic process significantly simplifies the state space. In reality, workers gradually accumulate firm-specific skills over time. However, modeling in this way would require the model to keep track of the quantity of workers at each tenure for each firm. At the firm level, gradual accumulation of firm-specific skills and probabilistic transition are equivalent. In Section 1.6.3, we document that a probabilistic transition from junior to senior worker does a reasonable job of approximating the wage profile across worker tenure.

In our model, junior workers hired in the past who have not become senior are identical to a new hire who has never worked for the firm. Consequently, firm's decisions are independent of the number of junior workers employed in prior periods and the state space reduces to the number of senior workers for each firm.

Timing

Firms enter each period with n^S senior workers and decide how many junior workers n^J to hire. Working capital is required to cover the cash flow mismatch between wage payment of $\left(w^S n^S + w^J n^J\right)$ at the beginning of the period and the realization of revenues. This working capital is financed by taking out an intra-period loan from the financial intermediaries. The intra-period loan is repaid at the end of the period.

Next, productivity shock z is realized, production occurs, and revenue is generated according to z. If revenue is sufficient to pay back the financial intermediaries, firms pay back their debt, distribute the remaining profits as dividends to the households, and continue into the next period with $n^{S'} \equiv (1-\delta)\left(n^S + \zeta n^J\right)$ senior workers, where δ is the exogenous separation rate between firms and workers. If revenue is insufficient to pay back financial intermediaries, firms exit. Upon exiting, all revenue from the current period is confiscated by the financial intermediaries, and in the subsequent period, the owner starts a new firm with zero senior workers. In addition to exit caused by bankruptcy, firms also exit exogenously with some probability ξ each period.[11] A

capital receives hw^J in wages and provides $h\theta^J$ effective units of labor to the firm. When that worker turns senior, they receive a wage of hw^S and provide $h\theta^S$ units of effective labor to the firm. Finally, we normalize the average general human capital per household to 1 so that the total supply of labor remains unchanged. In this manner, our model implicitly can account for general forms of human capital without any additional modification.

[11] With a decreasing returns to scale production technology, the value of a firm without any senior

visual depiction of the timing is displayed in Figure 1.5.[12]

The price of the final output is normalized to 1 and wage rates are measured in units of real output.

A firm's hiring decision is most easily explained in the reverse order. For a firm with n^S senior workers, in order to determine how number junior workers n^J to hire, it needs to form expectations about the schedules of wage rates $w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right)$, interest rate $R\left(n^S, n^J\right)$, and the cutoff productivity level $z^*\left(n^S, n^J\right)$ for bankruptcy. We will start with the determination of these schedules, then return to the hiring decision of firms.

Wage Bargaining

Consider a firm with n^S senior workers which has chosen to hire n^J junior workers. To demonstrate wage bargaining, it is helpful to define the value functions at different points in time throughout a period. In particular, we denote the value functions at the beginning of the period before any hiring decisions are made by, V^1, S^1, and J^1 representing the value to a firm, senior worker, and junior worker, respectively. We denote the value functions after wage bargaining and the determination of interest rate and bankruptcy cutoff by, V^2, S^2, and J^2.[13] Figure 1.6 illustrates the timing for the

workers is strictly positive, capturing the economic rent derived from the fixed factor. Implicit in our model is the assumption that financial intermediaries are unable to seize and liquidate this fixed factor. One interpretation for this fixed factor is the managerial human capital provided by the owner of the firm. Upon default, lenders are unable to confiscate the value provided by this human capital and consequently firms are unable to borrow against it or use it pay their liabilities.

[12] We assume that at the end of each period, firms pay out all profits (if any) as dividends. That is to say, firms are restricted from retaining earnings. There exists a large literature in finance which argues that there are substantial costs of maintaining a large buffer stock of retained earnings. For example, Jensen (1986) argued that, in practice, if firms retain a large amount of their earnings in order to build up a buffer, managers use these funds in ways that benefit their private interests rather than shareholder interests. Since shareholders understand this, they give managers incentives to pay out funds immediately rather than retain them. We crudely model this effect by preventing firms from retaining any earnings in the form of cash. However, firms do save in a sense by investing in junior workers at a loss which is earned back when the workers become senior. In addition to higher productivity, senior workers also serve as protection against default in the case of a poor realization of z. Mature firms default significantly less than their younger counterparts. For brand new start-up firms, this assumption on retained earnings makes no difference in the first period. But it slows down firm hiring in subsequent periods. Allowing retention of earnings in a liquid form weakens our result quantitatively but not qualitatively.

[13] Note that V^2 reduces to V^1 when it is evaluated at the equilibrium policy function and wage schedules. The same applies for S^2 and J^2.

various value functions.

Suppose the wage rates for senior and junior workers have been determined to be w^S and w^J, and the interest rate and bankruptcy cutoff have been determined to be R and z^*. Define the state variables at this point in time by $\boldsymbol{s} \equiv \left(n^S, n^J, w^S, w^J, R, z^*\right)$. Then the value functions to the firm, its senior workers, and its junior workers are:

$$
\begin{aligned}
V^2(\boldsymbol{s}) &= \int_{z^*} \left[z\left(\theta^S n^S + \theta^J n^J\right)^\alpha - \left(w^S n^S + w^J n^J\right) R + \beta\left(1-\xi\right) V^1\left(n^{S'}\right) \right] f(z)\, dz \\
&\quad + \beta\left[1 - \left(1 - F(z^*)\right)\left(1-\xi\right)\right] V^1(0) \\
S^2(\boldsymbol{s}) &= w^S + \beta \underbrace{\left(1-\delta\right)\left(1 - F(z^*)\right)\left(1-\xi\right)}_{\text{No Separation from the Firm}} S^1\left(n^{S'}\right) \\
&\quad + \beta \underbrace{\left[1 - \left(1-\delta\right)\left(1 - F(z^*)\right)\left(1-\xi\right)\right]}_{\text{Separation from the Firm}} J^1 \\
J^2(\boldsymbol{s}) &= w^J + \beta \underbrace{\left(1-\delta\right)\left(1 - F(z^*)\right)\left(1-\xi\right)\zeta}_{\text{No Separation and Transition into Senior}} S^1\left(n^{S'}\right) \\
&\quad + \beta \underbrace{\left[1 - \left(1-\delta\right)\left(1 - F(z^*)\right)\left(1-\xi\right)\zeta\right]}_{\text{Separation or Not Transition into Senior}} J^1
\end{aligned}
$$

where $n^{S'} \equiv (1-\delta)\left(n^S + \zeta n^J\right)$ is the number of senior workers at the firm tomorrow conditional on survival.

The value to the firm consists of current period profits, as well as a continuation value depending on survival. If the productivity shock z is above the default cutoff z^* and exogenous firm exit does not occur, firms survive into the subsequent period with $n^{S'} \equiv \left(n^S + \zeta n^J\right)$ senior workers. Otherwise, firms exit and the owners start new firms in the following period with no senior workers.

The value $J^2(\boldsymbol{s})$ to a junior worker consists of the current period wage they receive w^J plus a continuation value. If the firm survives and there is no exogenous separation of the worker from the firm, then the junior worker transitions into a senior worker with probability ζ in the subsequent period. Otherwise, they stay junior.

The value $S^2(\boldsymbol{s})$ to a senior worker is similar. The only difference is that senior workers stay senior with probability one, unless either the firm exits or exogenous separation occurs.

With these value functions defined, we can now analyze wage determination.

There is a competitive labor market for junior workers. The wage rate for junior workers needs to satisfy the market clearing condition:

$$J^2\left(n^S, n^J, w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right), R\left(n^S, n^J\right), z^*\left(n^S, n^J\right)\right) = J^1. \tag{1.1}$$

Intuitively, junior workers have the freedom of choosing which firm to work for. Therefore, firms of all sizes need to offer a common market value J^1 to make junior workers indifferent about which firm to join. Note that Equation (1.1) does not simply imply an equalization of junior wage across all firm sizes. This is because firms of different sizes have different probabilities of survival. Firm exit leads to the separation of workers from firms. In this case, junior workers lose the opportunity of becoming senior and must start over as junior at another firm. Therefore, firms with higher probability of default need to offer a higher junior wage w^J to compensate junior workers for the risk.

The wage rates for senior workers are determined through bargaining. Due to firm-specific skills, there exists joint surplus when a senior worker stays with their firm. If this match is destroyed, both the firm and the senior worker become worse off: the firm has to replace the senior worker with unskilled junior workers; and the senior worker has to start over at another firm as a junior worker. We assume that the match surplus between a firm and its senior workers is shared via bargaining. In particular, the surplus to a senior worker of staying with their firm is $S^2(\boldsymbol{s}) - J^2(\boldsymbol{s})$. The surplus to a firm of being matched with the marginal senior worker is $\frac{dV^2}{dn^S}$.[14] We assume that a fraction ϕ of the joint surplus is obtained by senior workers:

$$\frac{\left\{\begin{array}{c} S^2\left(n^S, n^J, w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right), R\left(n^S, n^J\right), z^*\left(n^S, n^J\right)\right) \\ -J^2\left(n^S, n^J, w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right), R\left(n^S, n^J\right), z^*\left(n^S, n^J\right)\right) \end{array}\right\}}{\frac{dV^2\left(n^S, n^J, w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right), R\left(n^S, n^J\right), z^*\left(n^S, n^J\right)\right)}{dn^S}} = \frac{\phi}{1-\phi}. \tag{1.2}$$

[14] Implicitly assumed about our bargaining protocol is that when a marginal senior worker threatens to quit a firm, the firm will "re-bargain" with all its remaining senior workers, junior workers (here, we think of the competitive determination of junior wage an extreme case of wage bargaining), as well as the financial intermediaries to re-determine the wage and interest rates. This re-bargaining process is captured by the second, third, fourth, and fifth terms in the following expansion of the total derivative:

$$\begin{aligned} &\frac{dV^2\left(n^S, n^J, w^S\left(n^S, n^J\right), w^J\left(n^S, n^J\right), R\left(n^S, n^J\right), z^*\left(n^S, n^J\right)\right)}{dn^S} \\ =&\frac{\partial V^2}{\partial n^S} + \frac{\partial V^2}{\partial w^S}\frac{\partial w^S}{\partial n^S} + \frac{\partial V^2}{\partial w^J}\frac{\partial w^J}{\partial n^S} + \frac{\partial V^2}{\partial R}\frac{\partial R}{\partial n^S} + \frac{\partial V^2}{\partial z^*}\frac{\partial z^*}{\partial n^S}. \end{aligned}$$

Our model is robust to other bargaining protocols.

Wage bargaining of senior and junior workers is assumed to happen simultaneously. Instead of treating senior and junior workers separately, we lump the two groups together into a single group of workers. We think of the competitive wage determination of junior workers as a special and extreme form of bargaining, where each junior worker automatically obtain the market value for junior workers J^1. With this generalized definition of bargaining, the simultaneous wage determination of senior and junior workers here is equivalent to firms bargaining with its workers on a one-by-one basis.

Our model also features a multi-party bargaining with decreasing returns. Many potential bargaining protocols are possible: all senior workers can form a union and bargain with firms as a whole, or senior workers can bargain with firms as small groups. Here we adopt the bargaining solution of Stole and Zwiebel (1996), in which firms negotiate with each of their senior workers in turn. This bargaining protocol was first used by Cahuc and Wasmer (2001), and later on by Hawkins and Acemoglu (2014) and Elsby and Michaels (2013) among others.

Financial Intermediaries

Competitive financial intermediaries make intra-period loans to firms. Since productivity shocks are idiosyncratic across firms, financial intermediaries are not subject to any aggregate uncertainty and behave as risk-neutral lenders. They offer an interest rate schedule R based on the number of senior workers n^S at a firm as well as the number of junior workers n^J hired by the firm. Interest rate schedule R and cutoff productivity level z^* for default are jointly determined by:

$$z^*\left(n^S,n^J\right)\left[\theta^S n^S+\theta^J n^J\right]^\alpha=\left(w^S\left(n^S,n^J\right)n^S+w^J\left(n^S,n^J\right)n^J\right)R\left(n^S,n^J\right) \tag{1.3}$$

$$\begin{aligned} w^S\left(n^S,n^J\right)n^S+w^J\left(n^S,n^J\right)n^J=&\int^{z^*\left(n^S,n^J\right)} z\left[\theta^S n^S+\theta^J n^J\right]^\alpha f(z)dz \\ &+\int_{z^*\left(n^S,n^J\right)}\left(w^S\left(n^S,n^J\right)n^S+w^J\left(n^S,n^J\right)n^J\right)R\left(n^S,n^J\right)f(z)dz \end{aligned} \tag{1.4}$$

where Equation (1.3) defines the cutoff level in productivity draws z^* below which firms are unable to repay the loan and Equation (1.4) is the break-even condition for risk-neutral financial intermediaries. Intuitively, firms with a higher risk of default need to pay a higher interest rate on their working-capital loan to compensate financial intermediaries for the lower probability of getting paid back.

Implicitly assumed here is a lack of insurance for firms. Financial intermediaries are not allowed to sign long term contracts with firms to insure against their idiosyncratic productivity risk. This lack of insurance can arise when there is asymmetric information between firms and financial intermediaries. For example, if the productivity shocks are unobservable by financial intermediaries so that no contracts can be contingent upon the shock realizations. Here this lack of commitment is assumed for simplicity. This is more natural for small firms which have not built up a high credit score, or established any reputation or relationship with banks.

Equations (1.1), (1.2), (1.3), and (1.4) provide us with four equations and four unknowns: $w^S\left(n^S,n^J\right)$, $w^J\left(n^S,n^J\right)$, $R\left(n^S,n^J\right)$, and $z^*\left(n^S,n^J\right)$. With these four schedules determined, we can proceed to the hiring decision of firms.

Firm Hiring Decision

Firms solve the following Bellman equation:

$$V^1(n^S) = \max_{n^J} \quad V^2\left(n^S,n^J,w^S\left(n^S,n^J\right),w^J\left(n^S,n^J\right),R\left(n^S,n^J\right),z^*\left(n^S,n^J\right)\right) \tag{1.5}$$

Lemma 1 *Combining the Bellman's equation of firms with the break-even condition of financial intermediaries, we can simplify Bellman Equation* (1.5) *to:*

$$\begin{aligned} V^1(n^S) = & \max_{n^J} E[z]\left[\theta^S n^S+\theta^J n^J\right]^\alpha - \left(w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J\right) \\ & +\beta\left[1-\left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)(1-\xi)\right]V^1(0) \\ & +\beta\left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)(1-\xi)V^1\left(n^{S'}\right) \end{aligned}$$

Intuitively, since financial intermediaries are risk-neutral and breaking even, it is as if firms are maximizing expected profits for the current period. The effects of default only enter through the continuation value of firms. The proof of this lemma are relegated to Appendix A.3.

Wage bargaining compresses wages relative to the marginal productivity of labor. The following lemma provides an intuitive characterization of the wage bargaining protocol between senior workers and firms:

Lemma 2 *Assuming there is no risk of default or exogenous separation* $\xi=0$*, constant returns to scale* $\alpha=1$*, and one-period transition from junior to senior worker* $\zeta=1$*,* *wage bargaining Equation* (1.2) *implies:*

$$\left[w^S - w^J\right] \;=\; \phi\left[E(z)\,\theta^S - E(z)\,\theta^J\right]. \tag{1.6}$$

The proof of this lemma is in Appendix A.4. Intuitively, the wage difference between senior and junior workers is compressed relative to the difference in their productivities. The compression factor is equal to the bargaining power of senior workers ϕ. Firm-specific skills introduce a hold-up problem between senior workers and firms. When a worker transitions from junior into senior, their productivity increases from θ^J to θ^S. However, due to the hold-up problem, the worker is only able to bargain a fraction ϕ of their gain in productivity into an increase in their wage. The weaker the bargaining power ϕ of the worker, the less the gain in wage from junior to senior worker. In an extreme case where workers have no bargaining power $\phi = 0$, firms are essentially making take-it-or-leave-it offers to senior workers. In this case, senior workers get paid their outside option, which is the junior wage. This wage compression is visually displayed in Figure 1.7.

There are two implications from wage bargaining Equation 1.6. First, the wage of senior workers relative to junior workers can be used to back out the differences in productivity levels across workers. Second, senior workers are more valuable to a firm relative to junior workers due to wage compression. In equilibrium, competition in the market for junior workers ensures that firms break even over the lifetime of a worker. Hiring a junior worker is a costly investment which pays off when the worker becomes senior. In this sense, senior workers represent a real value to the firm. Should the firm fail to survive, the loss of senior workers represents a real cost. For this reason, firms are cautious in their hiring in order to balance growth against the risk of losing their stock of senior workers.

This simple wage Equation (1.6) no longer holds when there is risk of default or decreasing returns to scale. With risk of default, senior workers provide an additional benefit to the firm, which is to mitigate the risk of default. This strengthens the bargaining position of senior workers. With a decreasing returns to scale production function, the marginal productivities of senior and junior workers are no longer θ^S and θ^J. Instead, they depend on the size of the firm.

1.4.3 Recursive Stationary Equilibrium

An abbreviated defintion of the recursive stationary equilibrium is provided here. A more detailed version is provided in Appendix (A.5).

Definition 1 *A recursive stationary equilibrium consists of: value functions for firms, senior workers, and junior workers; a policy function for firm's hiring decision; schedules of senior wage, junior wage, interest rate, and bankruptcy cutoff; and a distribution of firm sizes, aggregate output, and labor such that:*

1. *Give wage and interest rate schedules, the value function of firms solves the firm's Bellman equation*
2. *Schedules for wage rates, interest rate, and bankruptcy cutoff are jointly determined through wage bargaining and intermediary break-even equations:* (1.1), (1.2), (1.3), *and* (1.4)
3. *The labor market clears*
4. *The distribution of firms is consistent with the policy function*

1.5 Stationary Equilibrium

In this section, some intuition is provided about our cautious hiring mechanism. In particular, we demonstrate how the productivity gap θ^S/θ^J between senior and junior workers affects firm growth and the speed of employment recovery following a recession.

The cautious hiring behavior of firms comes from the combination of firm-specific skills and the lack of commitment by firms. Firm-specific skills give firms ex-post monopoly power and leads to a compressed wage profile relative to productivity profile. Junior workers are overpaid relative to their productivity and senior workers are underpaid. Hiring junior workers is risky because junior workers lower the expected current period profit and increase the probability of bankruptcy. Bankruptcy leads to the separation of firms from their stock of senior workers. Since these workers provide positive surplus, bankruptcy is costly.

Firms enter each period with a stock of senior workers who produce a positive expected cash flow. This positive cash flow is used to compensate for the expected losses of hiring junior workers. Junior workers that are retained have some probability of becoming senior in the subsequent period. The risky nature of hiring junior workers makes firms cautious about hiring too quickly. As firms gradually accumulate senior workers, they will eventually reach a mature size due to the decreasing returns to scale technology. At this point, firms hire just enough junior workers each period to compensate for the exogenous separation of senior workers.

Next we analyze how a change in θ^S/θ^J affects the rate of growth for firms. Equilibrium hiring decisions are chiefly dependent on the ratio of θ^S/θ^J rather than the level of each.[15] To simplify the intuition in this section, we fix θ^J at 1 and only analyze changes in θ^S.

Consider a comparative statics exercise. We simulate three economies, one without firm-specific human capital $\left(\theta^S = \theta^J = 1\right)$, one with a lower importance of firm-specific human capital, θ^S_{low}, and one with a higher importance of firm-specific human capital, θ^S_{high}. Specifically $1 < \theta^S_{low} < \theta^S_{high}$. We plot and compare the growth paths of the three economies in their respective stationary equilibria.

Figure 1.8 is a plot of the growth path for a typical firm in each of the three economies. The horizontal axis measures the number of senior workers n^S a firm enters the period with. The vertical axis measures the number of senior workers the firm will have at the beginning of the subsequent period (specifically, it is equal to the number of senior workers, n^S, plus the number of junior workers hired, n^J, after accounting for those workers exogenously separated from the firm, $(1-\delta)\left(n^S + \zeta n^J\right)$). In the case where senior and junior workers are equally productive (left panel, $\theta^S = \theta^J = 1$), the optimal size of a firm is reached in one period no matter how many senior workers the firm starts with. This is because junior workers are as productive as senior workers so they no longer incur an initial investment upon hiring (nor do senior worker offer a positive expected return in this setting). In this case, the hiring decision becomes a static problem. Firms always hire the optimal number of workers in each period. When we introduce a difference in the productivity levels between senior and junior workers (middle panel, θ^S_{low}), the optimal size of a firm is gradually reached as the firm balances growth against risk of bankruptcy. A further increase in the productivity gap between

[15] Suppose θ^S and θ^J are both scaled by λ. The equilibrium wage rates will rise by λ^α. None of the firms' hiring decisions will be affected.

θ^J and θ^S (right panel, θ^S_{high}) causes start-up firms to take even smaller steps in terms of hiring. As a result, it takes more steps (a longer time) for firms to reach the optimal size.

To understand how exactly an increase in θ^S/θ^J leads to slower growth of firms, we decompose the change in firm's growth path into a partial equilibrium effect and a general equilibrium effect. Following an increase in θ^S, and holding wage rates fixed, firms are reaping more surplus off each senior worker they employ. As a result, firms increase their hiring of junior workers who will become senior workers later on. This partial equilibrium effect tends to speed up firm growth.[16] At the same time, an increase in the demand for junior workers drives up wage rates. This general equilibrium effect tends to discourage firm hiring and slows down firm growth.

To separate these two effects, we conduct a counterfactual experiment. As we transit from one stationary equilibrium with θ^S_{low} to another with θ^S_{high}, we decompose the change in the policy functions into partial and general equilibrium effects. In Figure 1.9, we plot the respective policy functions. The blue curve is the policy function for the stationary equilibrium with θ^S_{low}, and the red curve is the policy function for the stationary equilibrium with θ^S_{high}. The pink curve is a counterfactual policy function for an economy with θ^S_{high} but with wage and interest rates for both senior and junior workers fixed at the levels in the equilibrium with θ^S_{low}.[17]

The partial equilibrium effect is represented by the shift from the blue curve to the pink curve. Following an increase in θ^S (holding wage and interest rates fixed), senior workers become more valuable to firms. As a result, firms increase their hiring of junior workers due to an increase in the future value of a worker. This partial equilibrium effect is heterogeneous across firms. Specifically it is weaker (or even negative) for small firms relative to large firms. As θ^S increases, the positive cash flow each senior worker brings to the firm increases. This lowers the probability of default. Compared with large firms, small firms have fewer senior workers. Consequently, the default probability for small firms does not decline as much. As a result, small firms do not increase their hiring of junior workers as much as large firms.

[16] Note that there are multiple distinct forces which together comprise the complete partial equilibrium effect. This is explored further below.

[17] Since wages aren't permitted to adjust in this counterfactual experiment, the labor market does not clear for the green line.

To illustrate the heterogeneity of the partial equilibrium effect, Figure 1.10 plots the default probabilities as a function of the number of junior workers hired. The left, middle, and right panels correspond to firms of sizes 0, 5, and 15 respectively (i.e., having 0, 5, and 15 senior workers). The solid blue curves plot the default probabilities in an economy with $\theta^S = \theta^S_{low}$. The circles on each solid curve indicate the optimal hiring choices in the θ^S_{low} economy. Regardless of firm size, the more junior workers a firm hires, the higher the probability of default. Next, we increase the productivity level of senior workers from θ^S_{low} to θ^S_{high} while holding wage rates fixed at the equilibrium levels of θ^S_{low}. Following this increase, default probability curves shift down from the solid blue curves to the dashed red curves. There are significant reductions in the default probability for firms of sizes 5 and 15. Intuitively, an increase in the productivity of senior workers mitigates the default risk and makes junior workers less risky to hire. This allows firms of sizes 5 and 15 to hire junior workers more quickly while maintaining a similar level of default risk.[18] On the other hand, for firms of size 0, the default probability curve under $\theta^S = \theta^S_{high}$ coincides with the case $\theta^S = \theta^S_{low}$. Since these firms have no senior workers, there is no effect on their default probabilities when θ^S increases. Consequently, these start-up firms do not significantly alter their hiring of junior workers.[19]

[18] This can be seen in the middle and right panels. The optimal hiring under θ^S_{high} is higher than that under θ^S_{low} (squares are to the right of the circles), while the default probabilities do not increase much (squares and circles are roughly of equal heights).

[19] In fact, firms of size 0 reduce the hiring of junior workers. The Bellman equation for a firm of size 0 is:

$$\begin{aligned} V^1(0) &= \max_{n^J} E(z)\left[\theta^J n^J\right]^\alpha - w^J n^J + \beta V^1\left((1-\delta)\zeta n^J\right) \\ &\quad -\beta\left[1-\left(1-F\left(z^*\left(0,n^J\right)\right)(1-\xi)\right)\right]\left[V^1\left((1-\delta)\zeta n^J\right) - V^1(0)\right] \end{aligned}$$

As θ^S increases, the value function shifts upwards and becomes steeper. This has two opposing forces on the hiring decision. On the one hand, increased continuation value $V^1\left((1-\delta)\zeta n^J\right)$ encourages firms to hire more junior workers who will become senior workers in the following period, given that firms survive. On the other hand, increased continuation value $V\left((1-\delta)\zeta n^J\right)$ makes firm exit more painful (this is represented by a higher difference $\left[V^1\left((1-\delta)\zeta n^J\right) - V^1(0)\right]$). Since hiring junior workers increases the probability of exit $F\left(z^*\left(0,n^J\right)\right)$, this second force discourages firms from hiring junior workers.

For firms with a positive stock of senior workers, the Bellman equation is:

$$\begin{aligned} V^1\left(n^S\right) &= \max_{n^J} E(z)\left[\theta^S n^S + \theta^J n^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) + \beta V^1\left[(1-\delta)\left(n^S + \zeta n^J\right)\right] \\ &- \beta\left[1-\left(1-F\left(z^*\left(n^S,n^J\right)\right)(1-\xi)\right)\right]\left\{V^1\left[(1-\delta)\left(n^S+\zeta n^J\right)\right] - V^1(0)\right\} \end{aligned}$$

The general equilibrium effect is represented in Figure 1.9 by the change from the pink curve to the red curve. An increase in the wage rates is needed to clear the labor market as we move from the θ^S_{low} economy to the θ^S_{high} economy. This wage increase discourages firms from hiring junior workers. As a result, the red curve lies below the pink one. This general equilibrium effect reduces the hiring of firms across all sizes.

Combining the partial and general equilibrium effects (from the blue curve to the red curve), smaller firms with size close to zero reduce their hiring, while larger firms increase their hiring. As θ^S increases, a start-up firm of size zero will grow slowly initially, followed by more rapid growth later on. If we measure firm sizes as a fraction of the size of a mature firm, then growth slows down across all firm sizes.

Following a recession, a larger than normal fraction of firms goes bankruptcy which leads to a drop in aggregate employment. The recovery phase is mainly driven by owners of new firms starting from size zero and growing their firms to the mature size. A higher level of θ^S/θ^J slows down the growth rate of start-up firms and leads to a slower recovery in employment.[20]

1.6 Quantitative Analysis

Of crucial importance to our model is the contribution of firm-specific human capital to the productivity of labor. In this section, we provide an estimate of this parameter and analyze how it evolves over time.

In practice, disentangling the marginal contribution of a specific worker is quite difficult as production from most firms involves the coordinated effort of multiple individuals

The two forces above are still present. However, the second force (survival concern) is mitigated by an increase in θ^S for large firms $\left(F\left(z^*\left(n^S, n^J\right)\right)\right.$ shifts down as θ^S increases). Therefore, the first force dominates and it encourages firms to hire more junior workers.

[20] Recent work in related literature corroborates the idea that business cycle job growth is largely driven by young firms. Fort et al. (2013) seeks to reconcile apparent discrepancy between Moscarini and Postel-Vinay (2012) and Gertler and Gilchrist (1994) regarding whether large or small firms drive movements in employment throughout the business cycle. They find that firm age is the missing link to understanding differences across these papers and that it is young rather than mature firms that are primarily responsible for fluctuations over the business cycle both in downturns and upturns. Decker et al. (2014) provides additional support that it is young firms primarily responsible for job creation stating, "business startups account for 20% of US gross job creation while high-growth business (which are disproportionately young) account for 50% of gross job creation." Using data from the BDS, we compute that roughly four-fifths (79.8%) of net job creation in the years following the 2008 recession (2011 to 2014) was attributable to firms of age five or less.

completing various tasks both separately and in groups. Further, hiring decisions by firms are endogenous with output.[21]

Although output of individual workers is difficult to measure, wage rates for workers of various seniorities are observable. In Section 1.4, we hypothesize that the difference between the wage rates of senior and junior workers are determined through bargaining between senior workers and firms. Only a fraction of the gains in productivity is reflected in the wage. This gives rise to our calibration strategy for θ^S/θ^J: we estimate the wage return to seniority from the data, and calibrate θ^S/θ^J so that the average wage difference between senior and junior workers in our model matches that in the data.

Existing estimations of the return to seniority include Topel (1991), Altonji and Shakotko (1987), and Altonji and Williams (2005), among others. We extend the methodology of Topel (1991) and Altonji and Williams (2005) to more recent samples of CPS and PSID in order to compare the return to seniority before and after the mid-1980s.

1.6.1 Determining the Return to Seniority from CPS

We estimate the return to seniority following the methodology of Topel (1991). We employ data from the Current Population Survey using the Displaced Workers, Occupational Mobility and Job Tenure supplements. In this survey, we are able to identify workers who were displaced from jobs as a result of economic reasons (layoffs and plant closings). We use the loss in wages for workers who were displaced to discipline our estimation of the return to seniority.

Using data from the Displaced Worker supplement of the CPS data is advantageous for a number of reasons. First, experience and tenure tend to move together, making it difficult to identify the contribution of each. Since separation does not alter a worker's

[21] Still, there do exist some attempts in the literature to identify the gains in worker productivity from experience without relying on the wage data. One attempt at quantifying the gains to experience is Shaw and Lazear (2008). They study a firm which installs windshields where output can be quantified and directly linked to individual workers. A main finding is that there is a very steep learning curve over the first 8 months on the job (53%). Further, their data show that these output gains with tenure are not reflected in equal percentage pay gains: pay profiles are much flatter than output profiles in the first year and a half on the job. Installing windshields has a relatively easy learning curve. Still, there are substantial gains to tenure which are not associated with an equivalent gain in wage providing support that workers are a form of investment.

experience but does change tenure, we attribute changes in the wage prior to displacement and post displacement to the return to tenure.[22] Second, the CPS data permits us to limit the selection bias of separation. Workers were displaced from their jobs for exogenous reasons instead of endogenous ones such as incapability of workers or bad match quality between workers and jobs.

The average change in real wages after an exogenous job loss across all employees is displayed in Figure 1.11. The drop in wages is affected by the business cycle. Specifically, the survey conducted immediately following each recession yields a larger drop in wages than the average drop experienced in surrounding observations.[23] Panel A of Figure 1.11 includes all data points whereas Panel B of Figure 1.11 excludes the first point following each NBER recession. The series are overlaid with the average labor productivity data from Figure 1.2. The figure demonstrates a strong comovement between changes in the real wage and patterns in labor productivity over the business cycle. This provides suggestive evidence linking our mechanism and result.

The observed fall in wages for the single data point in 1973 was 2.75%. The average fall in wages across all subsequent points is 17.25%. If the points immediately following each recession are dropped, the fall in wages is 12.95%.

Using cross-sectional U.S. data, Christofides and Oswald (1992), Blanchflower et al. (1996), and Hildreth and Oswald (1997) estimate that a 1% increase in a firm's profitability leads to an increase in wages between 0.01% and 0.08%. Therefore, we choose a bargaining weight of $\phi = 5\%$ for senior workers.

From the average drop in wage and the bargaining weight above, we obtain an estimate for the relative productivity between senior and junior workers of $\theta^S/\theta^J = 1.55$ for periods prior to 1983, and $\theta^S/\theta^J = 4.45$ for periods after 1983 ($\theta^S/\theta^J = 3.59$ if points immediately following recessions are excluded).

Estimating the return to tenure from the Displaced Worker supplement of CPS is subject to several potential limitations. First, the loss in wage following a displacement may be due to factors other than the loss in seniority (e.g. scarring). This would

[22] Depending on the year of the survey, those interviewed are asked if they were displaced in the preceding 1 to 5 years. Consequently, we are unaware of the exact year of displacement. The fact that general experience tends to increase wages causes our estimate of θ^S/θ^J to be biased downwards both for older and more recent recessions.

[23] This has also been documented by Davis and Wachter (2011).

lead to an upward bias in our estimation of returns to tenure. Second, since wage loss can only be calculated for workers who found a new job after displacement, we are missing the wage loss for workers who fail to find a new job. This selection may lead to a downward bias in our estimation of the percentage loss in wage. Finally, the CPS Displaced Workers supplement contains relevant wage data only for years 1973, 1984, 1986, ..., up to 2010. As a result, 1973 is the only data point that can discipline our choice of θ^S/θ^J for periods prior to 1983.[24]

1.6.2 Determining the Return to Seniority from the PSID

Due to the limitations of the CPS data, we also estimate the return to seniority using the methods of Altonji and Williams (2005). We update their estimations using more recent waves from the PSID.

Altonji and Williams (2005) use an instrumental variable approach to estimate the returns to seniority from 1975 to 1991. Unlike the CPS Displaced Worker supplement, job changes in PSID were not entirely due to exogenous reasons. This introduces some endogeneity problems. The increase in wage following a one-year increase in tenure is potentially an upward-biased measure of the return to seniority due to selection. For example, a good worker, or a good match between worker and job leads to both longer tenure and higher wage payment.

Altonji and Williams (2005) use the demeaned tenure over time for each job match as the instrument for tenure.[25] Their IV1 estimator reported a 10-year return to seniority of 5.42% for the period from 1975 to 1982, and 13.91% for the period from 1983 to 1991. We extend the methodology of Altonji and Williams (2005) to include more recent waves of the PSID. Additionally, instead of using the 10-year return, we calculate the return to tenure using the average tenure of the sample period. Using our updated data and methodology, average return to tenure is 4.84% from 1972 to 1982 and 13.02% from 1983 to 2013. Combining these numbers with a bargaining weight of 5% implies a $\theta^S/\theta^J = 1.97$ for older recessions and $\theta^S/\theta^J = 3.60$ for newer recessions.

For our subsequent simulations, we select $\theta^S/\theta^J = 2$ for recessions prior to the

[24] Additional information is delegated to Appendix A.6.

[25] Details of their regression method are delegated to Section A.7 of the Appendix.

mid-1980s and $\theta^S/\theta^J = 3.5$ for recessions after.[26]

1.6.3 Parametrization

Our model includes 11 parameters in the stationary equilibrium and an additional parameter to determine the magnitude of the shock. The parameters in our model are selected to levels standard in the literature or calibrated to match certain moments in the data. Following Greenwood et al. (1988), we remove the wealth effect on labor supply by adopting the GHH preference of the form $u\left(c, L\right) = \frac{\left[c - \frac{L^{1+\nu}}{1+\nu}\right]^{1-\gamma} - 1}{1-\gamma}$.

Each period in our model represents a quarter. The household discount factor β is set to be 0.99 to match an annual interest rate of 4%. The risk aversion parameter γ is set to 3. Labor share in the production function is chosen to be $\alpha = 0.65$.

Business cycle models with endogenous labor supply usually have difficulty simultaneously matching the nearly acyclical movement of wage rates and the procyclical movement of employment. A labor elasticity consistent with the micro literature generates too little fluctuation in employment or too much fluctuation in the wage relative to what we observe in the data. In order to simultaneously align with these two facts, a larger labor elasticity is required. Here, we set $\nu = 0.2$ which implies an elasticity of labor supply equal to 5. If a lower elasticity of labor supply is selected, our model is still able to deliver a slower speed of employment recovery. However, it generates a smaller initial drop in employment than observed in the data. As an alternative to using a high labor elasticity, we can introduce wage rigidity which allows us to match these facts even with a perfectly inelastic labor supply.

ζ measures the probability of a junior worker acquiring firm-specific skills and transitioning into a senior worker. We select $\zeta = 0.125$ so it takes 2 years (8 quarters) on average for a junior worker to transition into senior. This is consistent with Altonji and Williams (2005) which mentions that "all the estimation methods suggest that the

[26] Our calibration of the ratio $\theta^S/\theta^J = 2$ for pre-1985 recessions is consistent with findings of Frazis and Loewenstein (2006) who use 1982 EOPP data to measure growth in productivity over the first two years of employment. They find that productivity of workers increases by 80% over the first two years, and that productivity growth is only partially reflected in wage growth.

In our model, all firm-specific human capital is lost upon separation. In reality, some firm-specific skills may be retained across job transitions (i.e. in the case of recall). Our identification strategy estimates only that portion of firm-specific human capital which is lost because any retained portion of firm-specific skills will not lead to a drop in wages.

return to seniority declines sharply after the first year or two". This also provides a fairly good approximation to our fitted return to tenure outlined in Section 1.6.2. In Figure 1.12, we plot the fitted return to tenure using the data from the PSID over the expected return to tenure in our calibrated model.

Table 1.2: Externally Calibrated Parameters

Param.	Description	Value	Source
β	Discount Rate	0.99	Annual Interest Rate 4%
γ	CRRA Parameter	3	Business Cycle Literature
α	Labor Share	0.65	65% Labor Share
ϕ	Bargaining Weight for Workers	0.05	Literature‡
ζ	Probability Junior Turns Senior	0.125	Altonji and Williams (2005)
ν	Inverse of Labor Elasticity	0.2	Elasticity of Labor Supply 5

‡Christofides and Oswald (1992), Blanchflower et al. (1996) and Hildreth and Oswald (1997).

We calibrate the remaining parameters of our model to reproduce a series of aggregate moments from the U.S. data. In particular, the exogenous separation rate between firms and workers is chosen to be $\delta = 10\%$ to match the average job destruction rate of continuing firms from 1977 to 2013 from the Business Dynamics Statistics data. This also results in an average employment spell of approximately 2.5 years across all employees which is consistent with Shimer (2005).[27] The firm-specific productivity z is i.i.d. across firms and over time following a log-normal distribution. The mean of the shock is calibrated to match an average firm size of 16. The standard deviation of the shock is calibrated to match the average one-year cumulative default rate of 2% (Standard and Poor's). Exogenous exit rate of firms ξ is calibrated to match the 10% average exit rate of firms as in Cooper and Haltiwanger (2006).

Selection of θ^S/θ^J and the bargaining weight ϕ were discussed in the previous section.

Our model focuses on the recovery following a recession. Recent work by Bloom et al. (2014) and Stock and Watson (2012) suggest that second moment rather than first moment shocks to TFP may be chiefly responsible for driving business cycle dynamics. Additionally, a negative TFP shock directly influences labor productivity, mechanically generating a procyclical pattern. In alignment with this literature and to disentangle

[27] To be more exact, our selection of δ results in an average employment spell of 2.56 years for older recessions and 2.90 years for more recent recessions.

Table 1.3: Internally Calibrated Parameters

Param.	Description	Value	Target	Model
δ	Exog. Separation Rate	8%	Avg. Emp. Spell 2.5 Years	2.5
μ	Mean of Prod. Shock	$5.6, 4.4$	Avg. Est. Size 16	16
σ	Std. Dev. of Prod. Shock	0.15	S&P 1-Yr Avg Cumu. Def. Rate 2%	2%
ξ	Exog. Exit Rate of Firms	8%	Avg Exit Rate 10% ‡	10%
θ^J	Junior Prod.	1	Normalization	
θ^S_{old}	Senior Prod. (pre-1985)	2	Ret. to Tenure 1972-1982 4.84%	4.9%
θ^S_{new}	Senior Prod. (post-1985)	3.5	Ret. to Tenure 1983-2013 13.02%	13.0%

‡ Cooper and Haltiwanger (2006).

our mechanism from the first moment TFP shock we simulate recessions using shocks that preserve the mean of TFP. In particular, in Section 1.7 recessions are simulated by a shock to the second moment of TFP (which results in a larger than normal fraction of firms to exit). These exiting firms are replaced by firms that start over from size zero and gradually grow back to the mature size, which is driving the recovery. As a robustness check, Appendix A.8 analyzes a simpler shock in which we exogenously force a fraction of firms to exit. The unexpected variance shock increase disproportionately causes a larger fraction of smaller firms to exit but both shocks feature similar business cycle dynamics. As a final exercise, in Appendix A.9, we simulate a recession using a temporary shock to the first moment of TFP.

1.7 Business Cycle Properties

Using the parameters from Section 1.6, we now compare the business cycle properties of output, employment, and average labor productivity generated by our model to those in the data. Specifically, we compare our model to the data for the largest recession in our sample period before and after the mid-1980s: the 1973 recession and the 2008 recession.

Our model accounts for the slower employment recovery between the 1973 and the 2008 recessions. It also qualitatively matches the procyclical pattern of average labor productivity for the 1973 recession and the countercyclical pattern for the 2008 recession.

1.7.1 Model Transition Problem

Our model focuses on the recovery following a recession. Recessions are simulated as a one-time, unexpected variance shock to z. This shock results in a larger than expected share of firms to exit who are separated from their senior workers. In the data, smaller (or younger) firms are more likely to exit during a downturn than larger (or older) firms. Modeling shocks as an increase in the second moment of z has the added benefit of endogenously aligning with this pattern. Recovery is driven by new firms replacing the exiting firms growing from size zero back to the mature size. We compare the recovery paths of output, employment and labor productivity of our model with those in the data.[28]

Let the shock happen in period $t = 0$. Assume that it takes T periods for the economy to return to the stationary equilibrium. The Bellman equation for firms is given by

$$\begin{aligned}
&V_t^1\left(n^S\right) = \\
&\max_{n^J} \int_{z_t^*(n^S,n^J)} \left\{ \begin{array}{c} z\left[n^J\theta^J + n^S\theta^S\right]^\alpha \\ -\left[w_t^S\left(n^S,n^J\right)n^S + w_t^J\left(n^S,n^J\right)n^J\right]R_t\left(n^S,n^J\right) \end{array} \right\} f\left(z\right)dz \\
&+\Lambda_{t,t+1}\left[1-\left(1-F\left(z_t^*\left(n^S,n^J\right)\right)\right)\left(1-\xi\right)\right]V_{t+1}^1\left(0\right) \\
&+\Lambda_{t,t+1}\left(1-F\left(z_t^*\left(n^S,n^J\right)\right)\right)\left(1-\xi\right)V_{t+1}^1\left(\left(1-\delta\right)\left(n^S+\zeta n^J\right)\right)
\end{aligned}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor for the household between period t and period $t + 1$.

Value functions during the transition are indexed by time t. This is because total output/consumption is changing over the transition which affects the stochastic discount factors $\Lambda_{t,t+1}$.

[28] As a robustness check Appendix A.8 analyzes a simpler shock in which we exogenously force a fraction of firms to exit. The unexpected variance shock increase disproportionately causes a larger fraction of smaller firms to exit but both shocks feature similar business cycle dynamics.

Our model focuses on the portion of business cycle dynamics driven by firm composition. A standard first-moment TFP shock calibrated to match first-period output loss without any shock to the higher moments does not generate sufficient disruption to firm composition to match business cycle dynamics of employment. In Appendix A.9 we use a first-moment TFP shock sufficiently large to match the drop in employment. This shock closely aligns with dynamics in the data for output, average labor productivity and employment from the second period onwards but significantly overshoots the drop in first-period output.

1.7.2 Transition Intuition

Slower Recovery in Employment

Following a recession, a larger fraction of firms exit relative to the stationary equilibrium. Consequently, there are more start-up firms and less mature firms. When the economy is recovering, young firms hire junior workers and gradually grow back to the mature size. Employment recovery follows.

When θ^S/θ^J is low, the difference between the productivity levels of senior and junior workers is low. Therefore, start-up firms grow fast, leading to a fast employment recovery. When θ^S/θ^J is high, start-up firms grow slowly, which leads to a slow employment recovery. The intuition and policy function comparisons are discussed in Section 1.5.

Change in the Cyclicality of Average Labor Productivity

Average labor productivity in our model is calculated as output per worker. Since senior and junior workers have different productivity levels, we can decompose the average labor productivity into two parts: output per effective unit of labor supplied, and the average effective labor per worker:

$$\begin{aligned}
ALP &= \frac{Y}{\text{Total Labor}} \qquad (1.7)\\
&= \frac{Y}{\text{Effective Labor Units}} \times \frac{\text{Effective Labor Units}}{\text{Total Labor}}\\
&= \underbrace{\frac{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right]^{\alpha} g\left(n^S\right) dn^S}{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right] g\left(n^S\right) dn^S}}_{\text{Decreasing Returns to Scale Factor}} \times \underbrace{\frac{\int_{n^S}\left[\theta^S n^S+\theta^J n^J\right] g\left(n^S\right) dn^S}{\int_{n^S}\left[n^S+n^J\right] g\left(n^S\right) dn^S}}_{\text{Worker Composition Factor}} \qquad (1.8)
\end{aligned}$$

The first term in Equation (1.8), Y/Effective Labor Units, (henceforth, decreasing returns to scale factor), measures how efficiently output can be produced with effective labor units. Due to the decreasing returns to scale technology, this efficiency is higher when total effective labor is smaller. The second term, Effective Labor Units/Total Labor, (henceforth, worker composition factor), measures the average amount of effective labor units provided per worker. It is a weighted average of θ^S and θ^J, with weights equal to the shares of senior and junior workers in the economy. The more senior workers there are, the larger this term will be.

During the transition, the decreasing returns to scale factor and the worker composition factor exert opposing forces on average labor productivity. On the one hand, after a recession, start-up firms desire to grow which results in a larger influx of junior workers. These junior workers provide a smaller amount of effective labor per worker than senior workers. This tends to drive down average labor productivity through the worker composition factor.

On the other hand, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium for two reasons. First, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in a smaller average firm size. Second, the shock causes a fraction of firms to exit. Owners of exiting firms start new firms of size zero and engage in cautious hiring. They are smaller than their fully-grown counterparts. Because of the decreasing returns to scale production technology, a smaller average firm size increases average labor productivity.[29]

When θ^S/θ^J is low, start-up firms grow fast by hiring a lot of junior workers. The composition effect from the lower productivity of junior workers outweighs the decreasing returns to scale effect. This results in a procyclical pattern of average labor productivity. When θ^S/θ^J is high, recovery is slower. Start-up firms are more cautious in their hiring of junior workers. The downward pressure of less productive junior workers is diluted across time. This causes the decreasing returns to scale force to dominate. As a result, a countercyclical pattern of average labor productivity emerges.

Figure 1.13 plots this decomposition of average labor productivity. The left panel corresponds to the fast recovery economy ($\theta^S/\theta^J = 2$), and the right panel corresponds to the slow recovery economy ($\theta^S/\theta^J = 3.5$). In each panel, the red line plots average labor productivity during the transition. It is decomposed into output per effective labor unit (decreasing returns to scale effect, blue line) and effective labor per worker (worker composition effect, green line). Comparing the worker composition effect of

[29] Our model abstracts from capital. Empirically, over the business cycle, the fluctuations in employment are not fully matched by changes in capital. Consequently there is more capital available per worker during recessions. This serves as our motivation for modeling production using a decreasing returns to scale technology in labor.

the slow recovery economy (right panel) with the fast recovery one (left panel), we observe that the effective labor per worker (green line) falls by significantly less. As outlined above, this is because firms are hiring junior workers more cautiously in the slow recovery economy. This slower hiring also results in a slower recovery of GDP. A more steady consumption path translates into less disruption of the stochastic discount factor and average firm size. All of this contributes to a weaker decreasing returns to scale effect. In aggregate, we observe that the worker composition effect outweighs the decreasing returns to scale effect in the fast recovery economy and vice versa in the slow recovery economy.

1.7.3 Comparing Model Predictions to the Data

Using the parameter θ^S calibrated in Section 1.6, we compute the recovery path of output, employment, and average labor productivity. For the 1973 recession we use $\theta^S = 2$, and for the 2008 recession we use $\theta^S = 3.5$.[30] The magnitude of the shock η is selected to match the percentage drop in output observed from the data.

Figure 1.14 compares the model output from the two different recessions. Since the magnitude of the recessions vary slightly, the employment series have been normalized for easier comparison. Specifically, the deviation of employment from the stationary equilibrium has been set to -100% for the first period of recovery. Normalizing in this manner allows us to compare the speed of recovery in employment across recessions. As can be observed in Figure 1.14, an increase in θ^S/θ^J causes a slower speed of recovery in employment. It also reverses the cyclicality of labor productivity. The causes for this are outlined in Section 1.7.2.

Next, we overlay the simulation results from our model with the actual data on output, employment, and labor productivity for the relevant recessions.

In Figure 1.15, we compare the time series of output, employment, and average labor productivity between our model and the data throughout the recession of 1973. Note that our model only uses a single parameter, η, to match the data to this particular recession. No parameter is used to target the speed of output and employment recovery.

[30] Here we use the 1973 recession and the 2008 recession as representatives for the older and newer recessions, respectively. For simulation results regarding other recessions, please refer to Section A.10 of the Appendix for details.

Instead, the speed of output and employment recovery is endogenously determined by the speed of firm hiring.

Our model does a reasonable job approximating the speed of recovery in output and employment, as observed in the left and middle panels of Figure 1.15, given that the only moment we target is the initial drop. In the middle panel of Figure 1.15, employment recovers by 1.3% (from -5.5% to -4.2%) in the data over the first year of recovery. Our model delivers 1.5% employment recovery (from 5.0% to 3.5%) over this window. In addition, in our model, employment recovers to the pre-recession level in approximately three years, which is similar to the data. A more detailed comparison of the recovery of employment between our model and the data is included in Table 1.4.

From the right panel of Figure 1.15, we see that following the 1973 recession, average labor productivity dropped with output in the data. Our model delivers this procyclical pattern in labor productivity. The intuition for this is as follows. In the 1973 recession, the difference between the productivity levels of senior and junior workers was relatively small. Therefore, start-up firms grew quickly. This fast growth in employment results in firms spending little time on the more productive portion of the decreasing returns to scale technology. However, our model suggests that average labor productivity in the economy was reduced following the recessions by the large influx of junior workers with lower productivity.

Figure 1.16 compares our model to the data for the 2008 recession. The only difference in the parametrization between the 1973 recession and the 2008 recession is a change in θ^S and the magnitude of the shock η. Again the model does a fairly good job at predicting the rate of recovery in output. It also generates a slower rate of recovery in employment due to the cautious hiring mechanism. Four years after the ending date of the recession, employment has not recovered to the pre-recession level, in both the data and our model.

In the 2008 recession, average labor productivity went up while output dropped. Our model also delivers this counter cyclical pattern in labor productivity (right panel of Figure 1.16). Intuitively, in 2008 the difference between the productivity levels of senior and junior workers is relatively large. Therefore, start-up firms grow slowly for fear of losing their senior workers upon default. This slow growth in employment dilutes the influx of junior workers over time and weakens the downward pressure of junior workers on average labor productivity. Slower hiring also allows firms to operate at a smaller average size and take advantage of the highly productive portion of the decreasing returns to scale technology. As a result, average labor productivity rises over the recovery path.

1.7.4 A Partial Explanation for Jobless Recovery

By normalizing actual employment data from the two recessions using the same method, we are able to calculate how much of the slower employment recovery can be explained by our mechanism. This comparison is displayed in Table 1.4. In this table, we calculate how much employment has recovered relative to its trough after one, two and three years. For example, employment reaches its trough at 5.5% below its pre-recession level in the 1973 recession. One year after the employment trough, employment is still 4.2% below its pre-recession level. We calculate the percentage recovery in employment over the first year as $(5.5\% - 4.2\%)/5.5\% = 24\%$. Calculations are included for both the data and our model.

Table 1.4: Comparison of Employment Recovery

	Years After Emp. Trough	Percentage Recovery	
		Model	Data
	1	39%	24%
1973 Recession	2	75%	44%
	3	89%	84%
	1	34%	20%
2008 Recession	2	65%	48%
	3	82%	58%

Employment recovery for the 1973 recession features a non-monotone path in the data. Specifically, employment drops to its trough in 1975 and recovers by 24% from 1975 to 1976. However, it suffers another drop in 1977 (observable in the middle panel of Figure 1.15). Our model is relatively simple and there is no mechanism in our model that delivers this “double-dip” feature. Still, our model provides a reasonable match to the three-year employment recovery statistic. Our model predicts an 89% recovery in three years which is close to the 84% recovery observed in the data.

For the 2008 recession, our model predicts 34%, 65%, and 82% cumulative employment recovery over the first one, two and three years respectively. This is faster than what we observe in the data (20%, 48%, and 58%). Over a three-year horizon, our model is only able to account for a portion of the slower employment recovery for the 2008 recession.

Table 1.5: Comparison of Employment Recovery

	Years After Emp. Trough	Percentage Recovery Model	Percentage Recovery Data	Portion Explained
1973 Recession	3	89%	84%	
2008 Recession	3	82%	58%	
Difference		7%	26%	27%

By measuring the change in the speed of recovery from 1973 to 2008 as the difference in the three-year percentage recovery rate, our change in θ^S/θ^J accounts for just over a quarter (7%/26% = 27%) of the change in the speed of recovery in employment between 1973 and 2008.

1.8 Conclusion

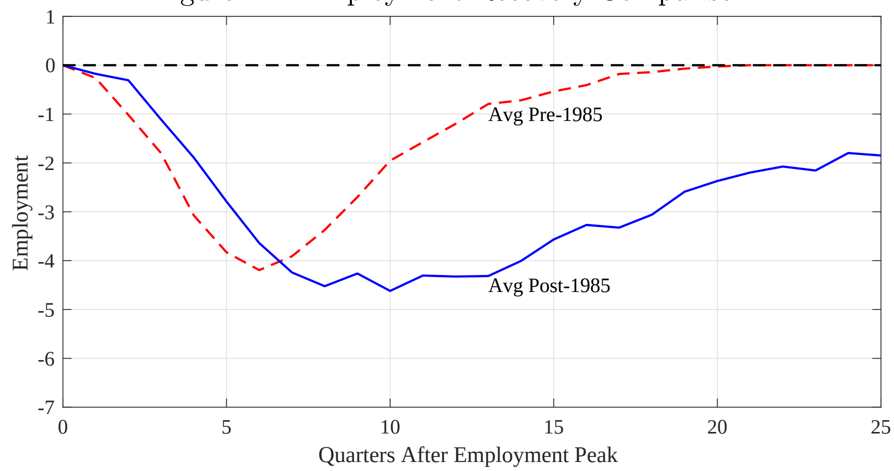
In this paper, we document two changes in the pattern of business cycles starting from the mid-1980s. First, the speed of recovery in employment has become significantly slower. Second, labor productivity has switched from being procyclical to acyclical or even countercyclical. We present a model that contributes to the explanation of both facts through the variation of a single parameter, namely the relative productivity of senior to junior workers.

In our model, when a recession occurs, many firms exit and new firms take their place. This results in two opposing forces on average labor productivity. First, start-up firms employ a disproportionately large number of junior workers, resulting in a decrease of average labor productivity. The second and countering force is that, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium. Due to the decreasing returns to scale production technology, a smaller average firm size tends to increase average labor productivity. Whether average labor productivity is procyclical or countercyclical depends on the relative strength of these two opposing forces.

In older recessions, the difference between the productivity of senior and junior workers is low so new firms tend to hire junior workers quickly. This concentrated influx of junior workers during periods immediately following a recession causes the downward force in average labor productivity to dominate. Further, since firms grow quickly, employment also recovers quickly.

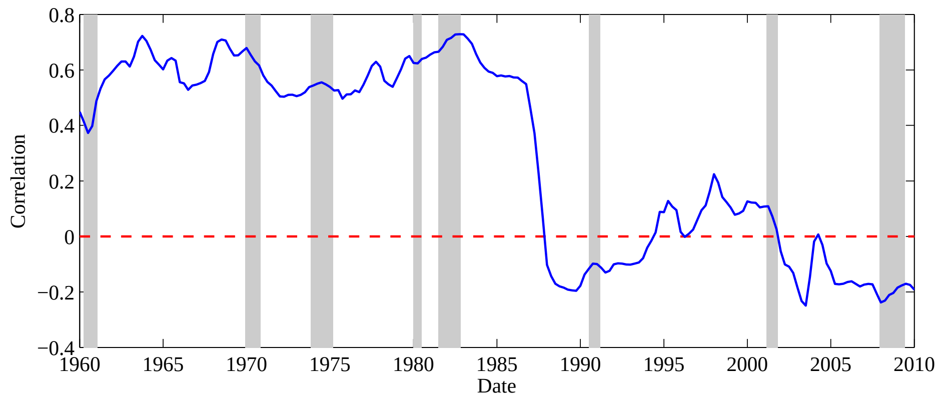
As the relative productivity of senior to junior workers increases, senior workers become more valuable to a firm. Consequently, firms are more cautious (slower) in hiring in order to avoid losing their stock of senior workers. This results in a slower recovery in employment. In addition to effects on employment, this slow hiring diffuses the downward effect on labor productivity across additional periods and results in firms spending more time on the relatively productive portion of their decreasing returns to scale technology. In this case, the upward force on average labor productivity dominates the downward force and results in the countercyclicality of labor productivity observed in more recent recessions.

Figure 1.1: Employment Recovery Comparison



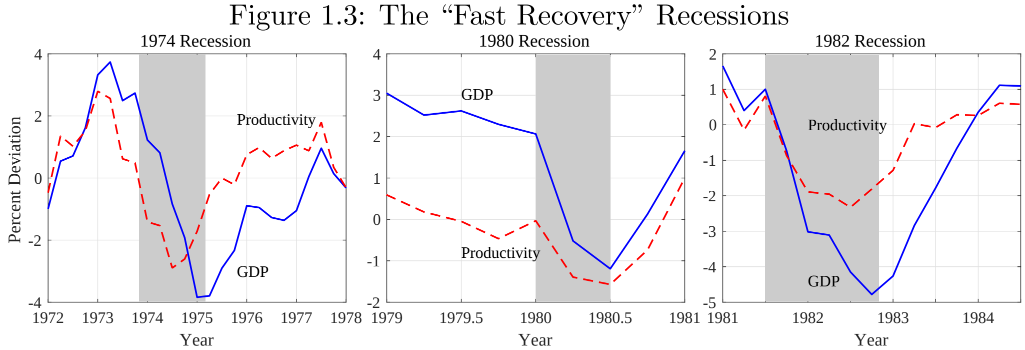
Source: Bureau of Labor Statistics, Authors' calculations
Pre-1985 recessions include 1969, 1973, and 1981 recessions

Figure 1.2: Ten Year Centered MA Correlation in Labor Productivity and GDP



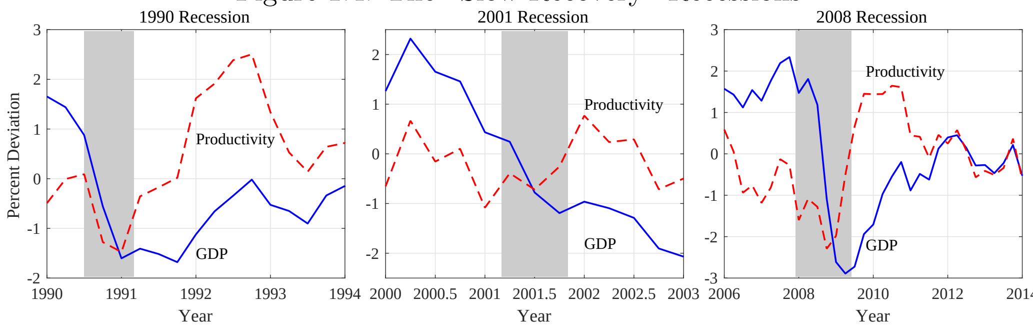
Source: Bureau of Labor Statistics, Bureau of Economic Analysis, Authors' calculations

Figure 1.3: The "Fast Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

Figure 1.4: The "Slow Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

Figure 1.5: Timing

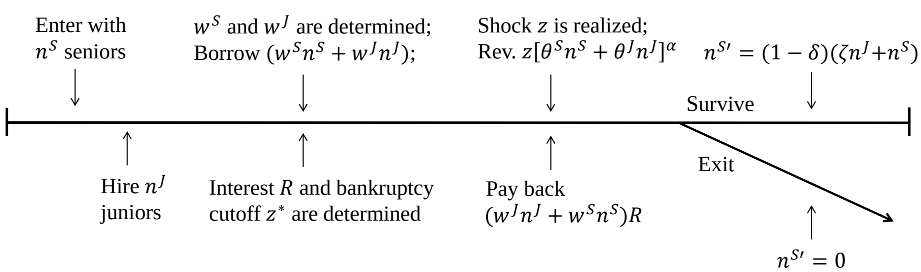


Figure 1.6: Timing with Value Functions

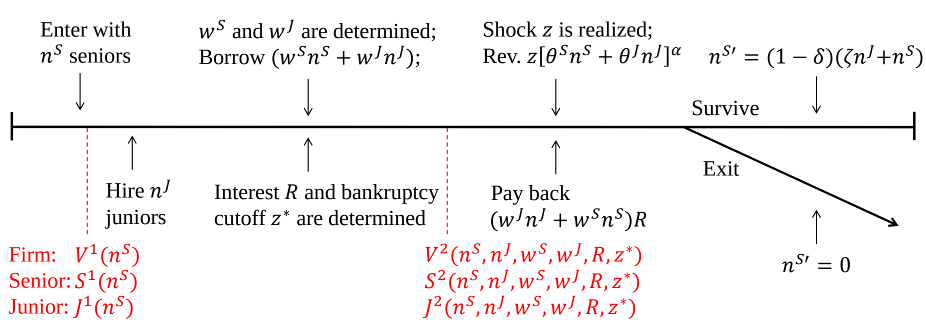


Figure 1.7: Wage Compression

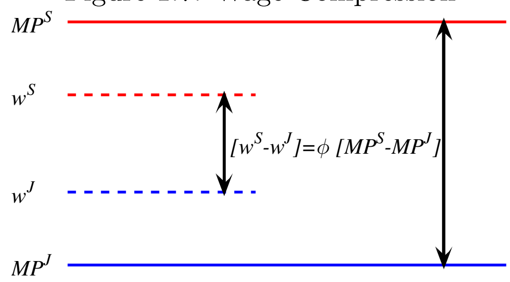


Figure 1.8: Firm Growth Paths for Different Productivity Gaps θ^S/θ^J

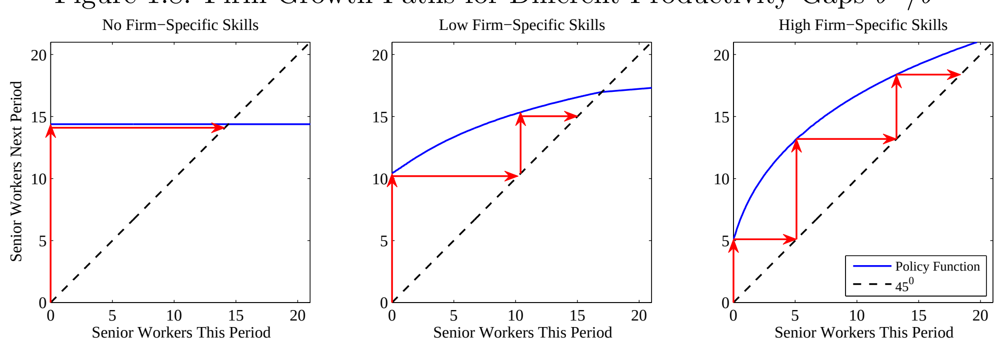


Figure 1.9: Partial and General Equilibrium Effects

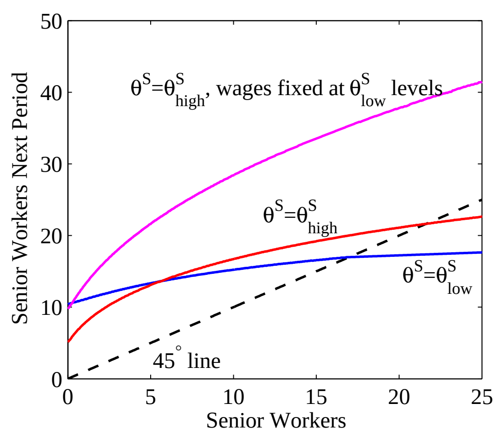
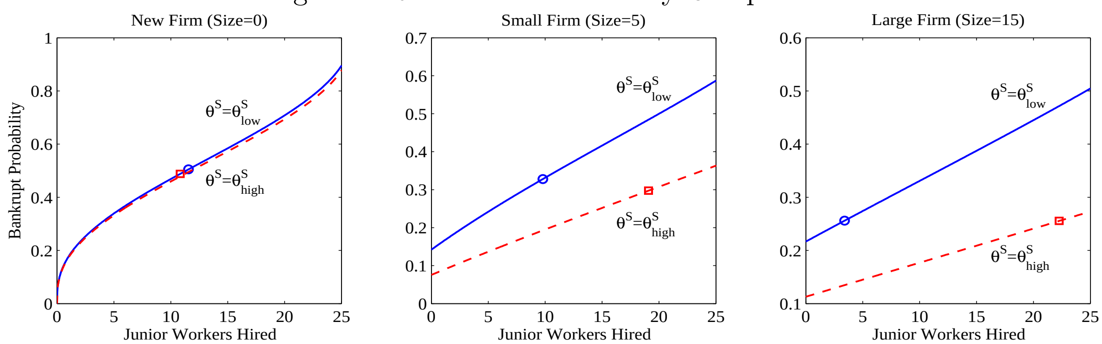
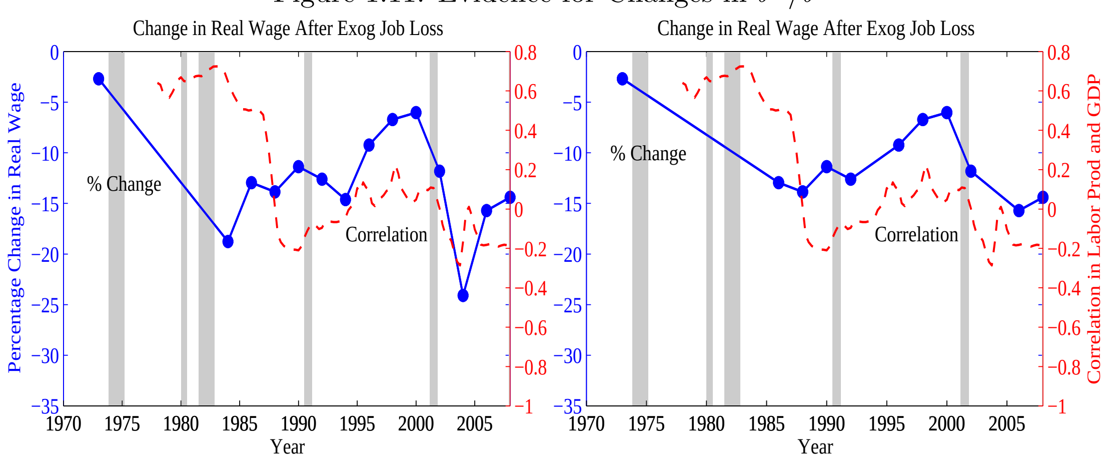


Figure 1.10: Default Probability Comparison



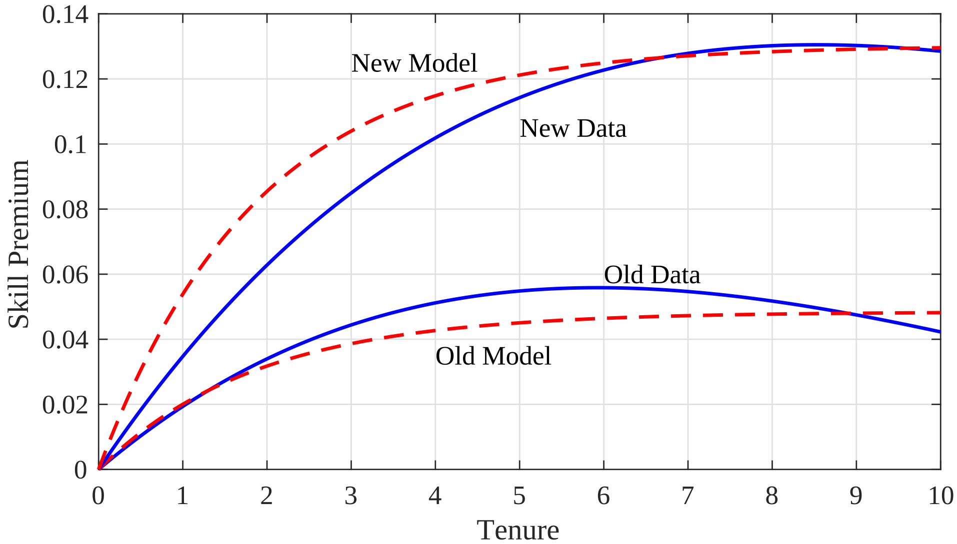
Circle: optimal junior hiring when $\theta^S = \theta^S_{low}$.
Square: optimal junior hiring when $\theta^S = \theta^S_{high}$.

Figure 1.11: Evidence for Changes in θ^S/θ^J



The blue line with circle markers is the percentage wage loss following a displacement. The red line is the correlation between average labor productivity and GDP.
Source: Displaced Worker's Supplement (CPS), Authors' calculations

Figure 1.12: Return to Tenure



Dotted red lines represent the expected return to tenure for workers as predicted by the model, the solid blue lines represent the fitted return to tenure as calculated from the PSID using the method of Altonji and Williams (2005).
Source: PSID, Authors' calculations

Figure 1.13: Decomposition of Average Labor Productivity (ALP)

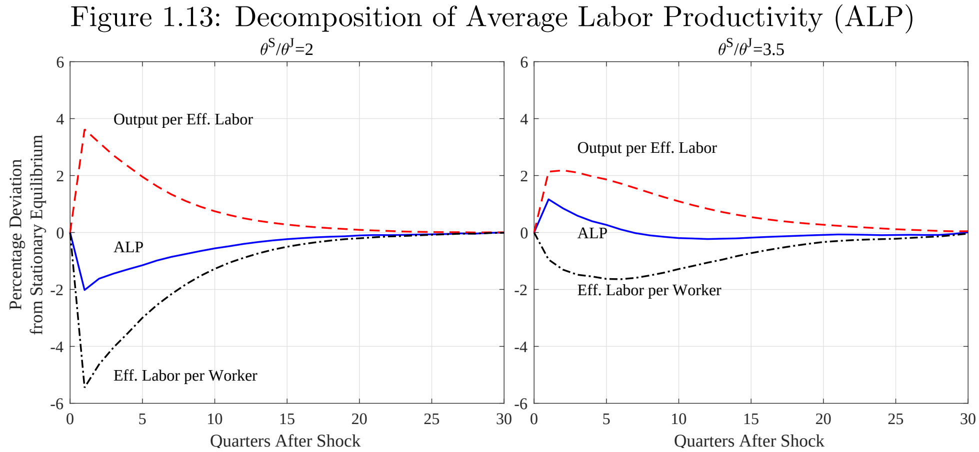


Figure 1.14: Transition Path Comparison of Models

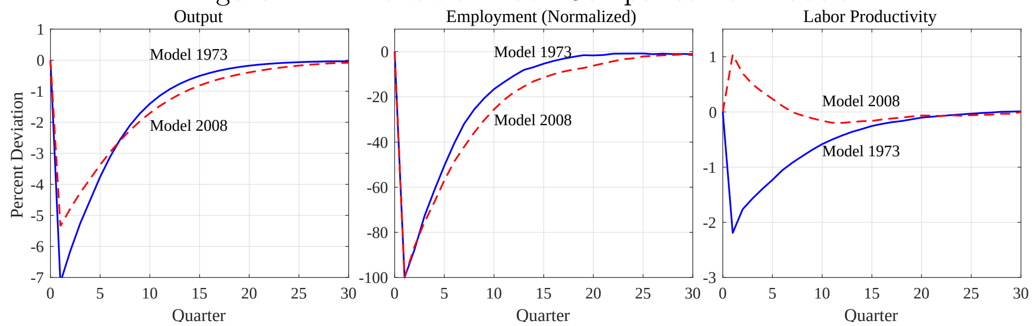
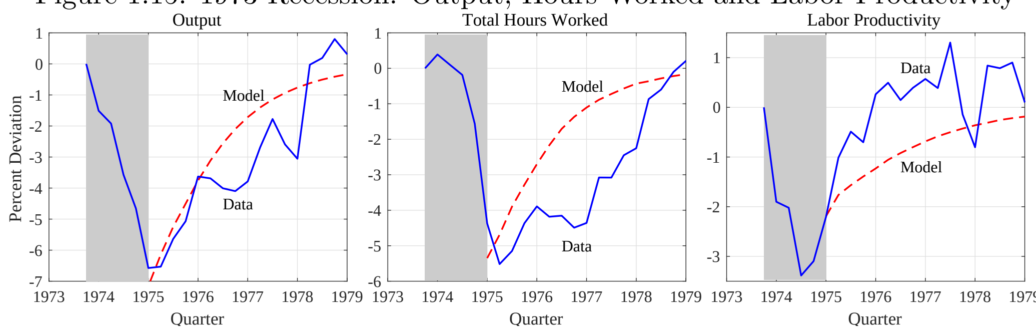
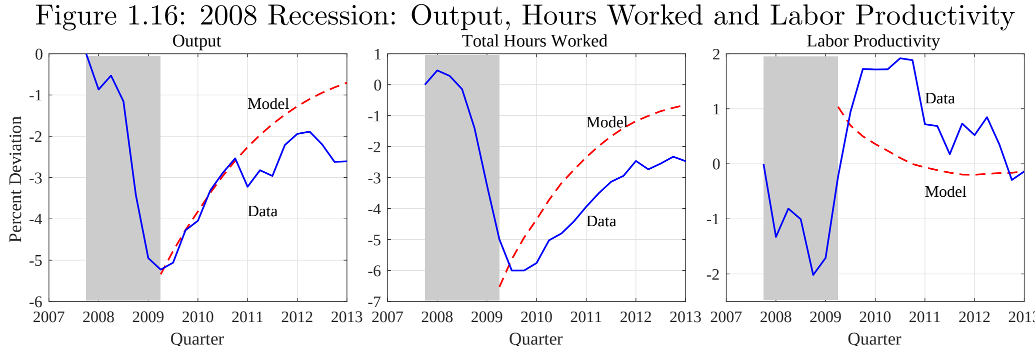


Figure 1.15: 1973 Recession: Output, Hours Worked and Labor Productivity



Source: Bureau of Labor Statistics, Authors' calculations

Figure 1.16: 2008 Recession: Output, Hours Worked and Labor Productivity



Source: Bureau of Labor Statistics, Authors' calculations

Chapter 2

How Do Financial Frictions Affect Self-Financing Firms?

2.1 Introduction

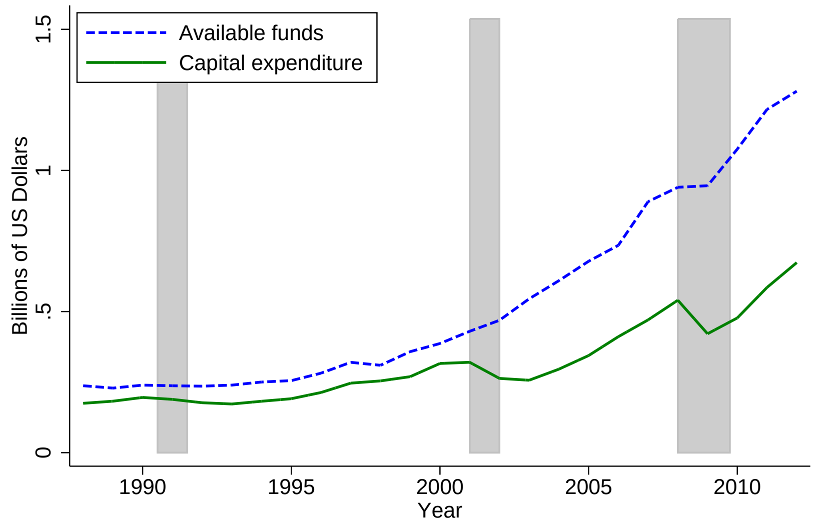
Our study of the Great Recession is motivated by three key observations about the investment and financing behavior of firms.

1. *Capital investment by self-financing firms fell significantly.* By self-financing firms we refer to firms that consistently generate more internal funds than capital expenditure. The green and blue lines in Figure 2.1 are the average capital expenditure and cash flow from operating activities of the largest 20% of COMPUSTAT firms in terms of asset level (henceforth, self-financing firms).[1] These self-financing firms generate higher net cash flows from operating activities than their capital expenditure, hence are capable of financing their investment without resorting to external funds. However, during the 2008 recession, their investment dropped by 24%. This pattern was also documented in Chari and Kehoe (2009) and Shourideh and Zetlin-Jones (2012).

2. *Liquid assets holdings by self-financing firms increased during the Great Recession, especially after the collapse of Lehman.* Figure 2.2 plots the time series of average

[1] These firms account for more than 30% of the civilian employment. Variables are taken from balance sheet and cash flow statements from COMPUSTAT.

Figure 2.1: Capital Expenditures and Available Funds for Self-Financing Firms



We measure available funds as net cash flow from operating activities.
Data source: COMPUSTAT and authors' calculation.

liquid assets holdings of self-financing firms. Liquid assets refer to the balance sheet item "cash and cash equivalents".[2] This surge in liquid assets holdings during the Great Recession is robust across sectors and firm sizes.

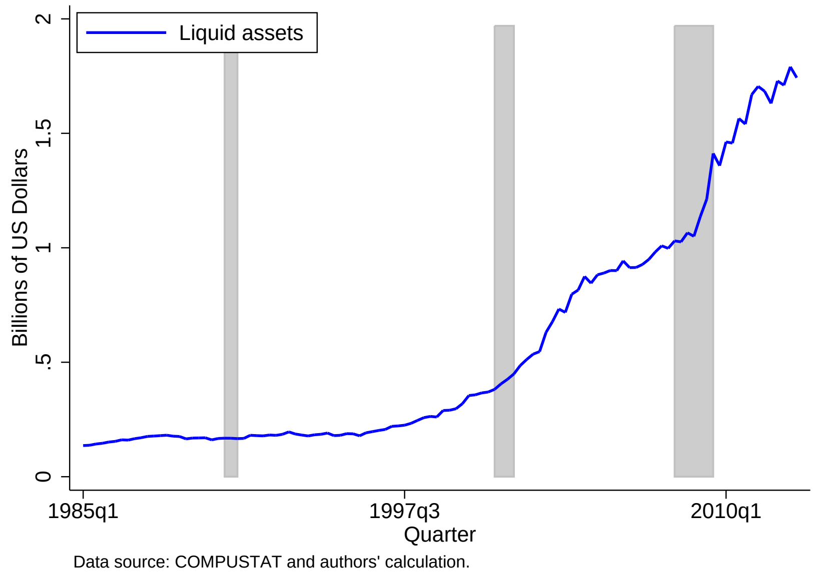
3. *Issuance of bank loans shrank while issuance of corporate bonds expanded.* Figure 2.3 is taken from Adrian et al. (2013). It plots the total issuance of bank loans and corporate bonds for all the COMPUSTAT firms (except financial and utility firms). We see a decrease in the issuance of bank loans (top panel) and an increase of bonds issuance (bottom panel) from the onset of the recession. This suggests that it may be worthwhile to model bank loans and corporate bonds as different debt instruments. An important distinction between the two is that bank loans are mainly in the form of revolving credit lines.[3]

These three facts cannot be simultaneously accounted for in models with collateral constraints or productivity shocks. In a standard model with financial frictions, a tightening

[2] Henceforth, we use liquid assets and cash interchangeably.

[3] Survey of Terms of Business Lending conducted by Federal Reserve Board reveals that 77% of C&I loans are made under commitment according to survey conducted on May 2014.

Figure 2.2: Liquid Assets Holdings for Self-Financing Firms

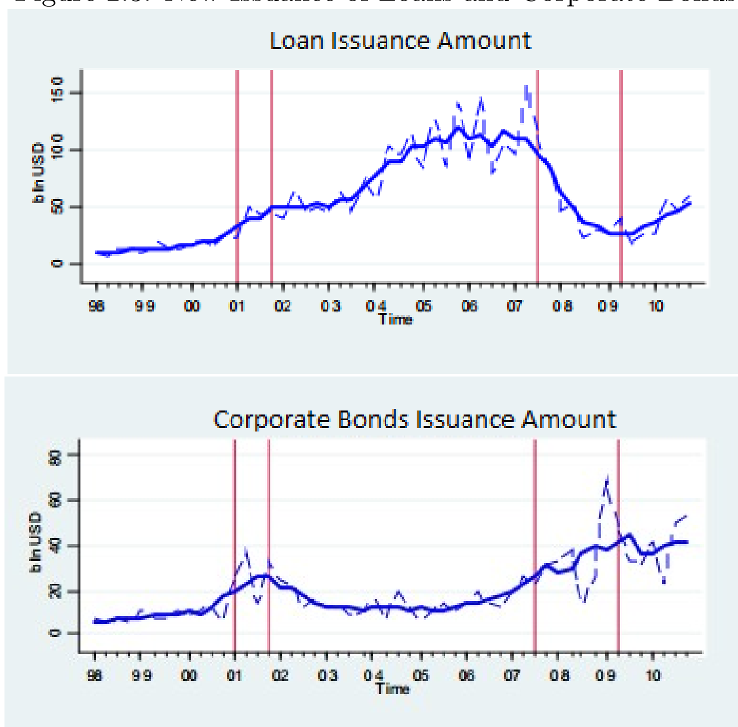


Liquid assets refer to the item “cash and cash equivalents (CHE)” from COMPUSTAT balance sheet data.

of the collateral constraint forces firms to reduce their capital investment. However, self-financing firms have enough internal funds to finance their capital expenditure, and do not have to resort to external funds. Therefore, it is not clear how a tightening of the collateral constraint would lead to reduced investment by these firms. In a standard RBC model, a negative productivity shock generates a drop in corporate investment. However, it does not have clear predictions about corporate liquid assets holdings. In fact, we show later that a negative productivity shock *reduces* firm's liquid assets holdings under the assumption that liquidity shocks are proportional to the level of capital stock, as in Holmstrom and Tirole (1998). Macroeconomic models with corporate borrowing usually ignore heterogeneity in corporate debt structure, hence cannot address the opposite behavior of bank loans and corporate bonds during the Great Recession.

To jointly account for the three observations above, we propose a model where capital investment is subject to *liquidity shocks*, which we model as unexpected cost that firms have to finance right away before their operations can be continued. As in Holmstrom and Tirole (1998), we interpret liquidity shocks as “cost overrun of the initial investment

Figure 2.3: New Issuance of Loans and Corporate Bonds



Top panel: new issuance of bank loans
Bottom panel: new issuance of corporate bonds.
Source: Adrian et al. (2013)

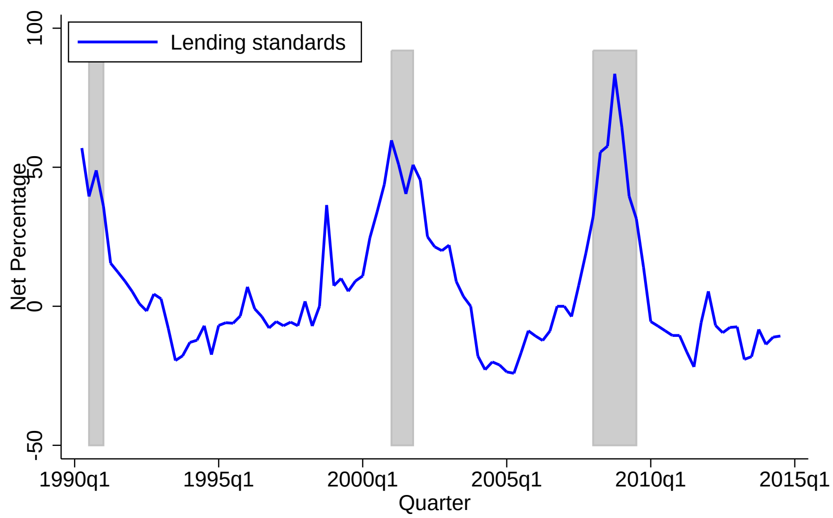
or a shortfall of revenue relative to operating expense during the intermediate period". Liquidity shocks have to be financed by either drawing down bank credit lines or selling liquid assets. In this environment, we study how an exogenous reduction of the credit lines available to firms (henceforth credit line shock) impacts their investment and cash holdings decisions.

The *intuition* why a credit line shock can simultaneously account for firm's investment and liquid assets holdings behavior is as follows. Bank credit lines and liquid assets are substitutes for insuring against liquidity shocks and safeguarding capital investment. Given the amount of credit lines available, firms have to bundle one unit of capital with certain units of liquid assets to cover the remaining liquidity risk. When banks tighten the credit lines, firms have to bundle extra units of liquid assets with each unit of capital to cover the increased liquidity risk. Holding liquid assets, however, is costly because of their low return. This increases the marginal cost of capital investment. Hence, firms optimally reduce their capital investment.

The assumption that credit lines serve as substitutes to cash holdings against liquidity shocks during the Great Recession is supported by the empirical finding in Almeida et al. (2014). In particular, they showed that at firm level “there is a strong negative correlation between cash holdings and credit lines in the crisis”.

The exogenous tightening of bank credit during the Great Recession is supported by the Senior Loan Officer Opinion Survey conducted by the Federal Reserve Board. As shown in Figure 2.4, the net percentage of domestic banks that have tightened their lending standards of commercial and industrial loans to large and middle-market firms increased from 25% at the beginning of 2008 to more than 75% at the peak of the recession.

Figure 2.4: Net Percentage of Domestic Banks that have Tightened Lending Standards



Here we measure lending standards as the net percentage of domestic banks tightening lending stand of commercial and industrial (C&I) loans to large and middle-market firms.

Data source: Senior Loan Officer Opinion Survey.

We calibrate the model to firm-level balance sheet information from COMPUSTAT and compare the transition dynamics of the model economy after a standard credit shock, a productivity shock, and a credit line shock. We find that:

1. A standard credit shock does not affect high net worth (hence unconstrained)

firms, leaving their capital expenditure and cash holdings unchanged.

2. A productivity shock generates a decrease of capital expenditure across firm sizes. But it implies a counter-factual reduction in firms' liquid assets holdings.

3. A credit line shock generates a decrease in capital expenditure as well as an increase in liquid assets holdings across firm sizes.

4. A credit line shock also generates a change in the composition of bank loans and corporate bonds as in the data.

The paper is structured as follows. Section 2.2 reviews literature. Section 2.3 presents a simple two-period example to illustrate our model trade-offs. Section 2.4 sets up the full-fledged recursive model. Section 2.5 studies the quantitative properties of our model. Section 2.6 concludes.

2.2 Related Literature

Our empirical findings are novel as we are the first to jointly document corporate financing, liquid assets holdings and corporate investment behavior for self-financing firms. Chari and Kehoe (2009) and Shourideh and Zetlin-Jones (2012) document that a large fraction of US corporations are able to self-finance their investment. There is also a large and growing literature studying why large corporations choose to hold more cash from 1980 on. For recent study on this issue, see for example Pinkowitz et al. (2012).

Our model is also related to the literature on financial frictions such as Khan and Thomas (2013) and Jermann and Quadrini (2012), or earlier ones like Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). This literature focuses on the role of bank debt as a source of external financing for investment. We focus instead on its role as revolving credit lines that provides firms with the flexibility of financing liquidity shocks. We show that adding this more realistic feature into the model helps to improve the model's cross-sectional predictions.

Our paper is also related to the literature on corporate liquidity management practices initiated by Holmstrom and Tirole (1998). We embed firm's liquidity management decision into a quantitative heterogeneous firms framework, and conduct numerical experiments to demonstrate that it can help us to think about corporate investment behavior of firms, especially during the Great Recession.

Lastly, our paper is related to a recent literature studying the implications of corporate debt structure on the macroeconomy, such as Fiore and Uhlig (2011), Fiore and Uhlig (2015) and Crouzet (2013). These papers study how firms choose between bank loans and corporate bonds. We study a different trade-off between bank loan and corporate bonds from these authors. Specifically, in our model bank loans are a more flexible source of financing that can be used to finance firm's liquidity shocks, as documented in Sufi (2009).

2.3 Model: Two-Period Example

In this section, we present a two-period example to explain the main intuition of our model.

Firms in the economy operate a decreasing returns to scale production technology, converting capital k into output $y = zk^{\alpha}$. The production technology of firms is subject to liquidity shocks as in Holmstrom and Tirole (1998). Specifically, after capital k has been installed but before output y is produced, a random additional cost of ρk has to be paid right away before production can continue. We interpret liquidity shocks as "cost overrun of the initial investment or a shortfall of revenue relative to operating expense during the intermediate period".

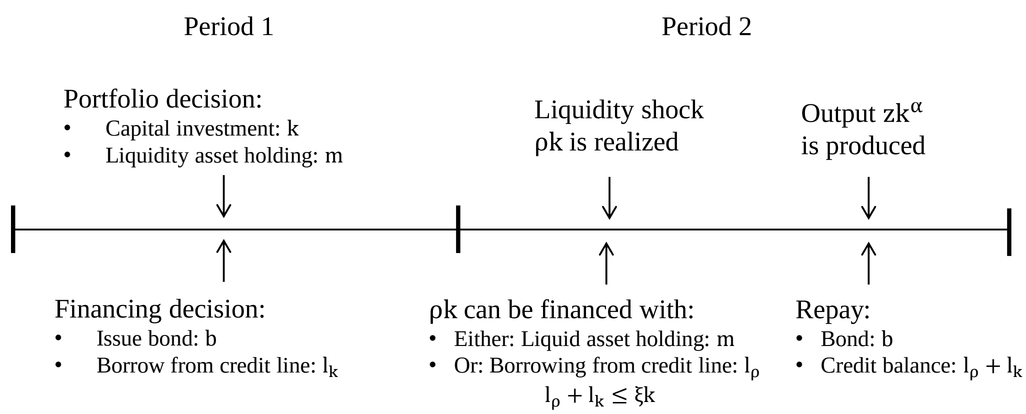
In addition to the decreasing returns production technology that is prone to liquidity shocks, firms also have access to a liquid saving technology with a constant return of one. This technology can be thought of as checking accounts at banks. At the end of each period, firms decide how to split their assets between capital investment k and liquid saving m.

Firms can issue bonds at interest rate R. They also have access to credit lines from banks. The credit lines are collateralized to the physical capital of the firms. The credit limit for a firm with capital stock k is ξk for some exogenous parameter ξ. Firms can finance the investment in capital by either issuing bonds b or drawing down their credit line. However, liquidity shocks can only be paid using credit lines or liquid saving m.

They cannot be financed using bonds. This is because bond issuance usually involves paperwork and preparations such as Security and Exchange Commission filings, which takes relatively long time to complete.

The timing of the model is as follows. In period 1, firms choose how much capital investment k to make. They also decide how much liquid saving m to make. Firms have 0 net worth (assume this for simplicity). All the investment and saving are financed by either issuing corporate bonds b or drawing down its bank credit line l_k.[4] On the morning of period 2, a liquidity shock[5] ρk is realized. It can be financed either with saving m or by drawing down bank credit line l_ρ.[6] In the afternoon, output y is produced. Firms make payment on corporate bonds and the used portion of their credit lines.

Figure 2.5: Timing (Two-Period)



Our model features both inter-period and intra-period borrowing. Corporate bond is issued in period 1 and repaid in period 2. Therefore, it carries an interest rate of R. Bank credit line can be used for either investment purpose or liquidity purpose. The

[4] Here the subscript k in l_k denotes that part of the credit line that is drawn down for capital investment purpose.

[5] A liquidity shock is an unexpected cost that firms have to finance right away before their operations can be continued. As in Holmstrom and Tirole (1998), we interpret liquidity shocks as "cost overrun of the initial investment or a shortfall of revenue relative to operating expense during the intermediate period".

[6] Here the subscript ρ in l_ρ denotes the part of the credit line that is drawn down to finance the liquidity shock ρk.

portion of the credit line used for investment purpose is an inter-period loan and carries an interest rate of R. The portion of the credit line used to pay the liquidity shock is an intra-period loan and carries an interest rate of one.

Firms solve the following problem:

$$\begin{aligned}
\pi(\xi) &= \max_{k,m,b,l_k,l_\rho} zk^\alpha - R\,(b+l_k) - l_\rho \\
\text{s.t.} \quad & m+k \leq b+l_k && (2.1) \\
& m+l_\rho \geq \rho k && (2.2) \\
& l_k+l_\rho \leq \xi k && (2.3)
\end{aligned}$$

Equation (2.1) is the budget constraint of firms: total investment in capital stock and liquid assets cannot exceed the total amount of funds firms are able to raise. Equation (2.2) is the liquidity constraint: liquidity shocks have to be financed by either liquid asset holding or bank credit line. Finally, Equation (2.3) is the credit limit constraint of firms: total draw-down of bank credit line for either capital investment or liquid shock financing cannot exceed firm's credit limit ξk.

To analytically characterize the solution to firm's problem, the following assumptions are made:

Assumption 1 *Inter-period interest rate R and liquidity shock ξ satisfy:*

1. $R > 1$*: inter-period borrowing rate is higher than intra-period borrowing rate.*

2. $\xi \leq \rho$*: credit line itself is not sufficient to cover the liquidity shock.*

Lemma 3 *Under Assumption 1, the optimal solution to firm's problem is characterized by:*

1. $m > 0$

2. $l_k = 0$

3. $l_\rho = \xi k$

4. $\alpha z k^{\alpha-1} = R(1+\rho) - (1-R)\,\xi$

That is, firm holds positive liquid asset, uses none of the credit line for financing capital investment, and saves the entire credit line for hedging the liquidity shock.

Intuitively, the liquidity shock has be to financed through either liquid asset holding m or bank credit line l_ρ. Liquid saving m carries an opportunity cost of $R > 1$ due to its low return. Therefore, firm minimizes their liquid asset holding and reserves the entire bank credit line for financing liquidity shock.

Proof 1 *From* (2.1) *and* (2.3) *of the problem, we get:* $m \geq \rho k - l_\rho \geq \rho k - \xi k + l_k > 0$. *Note that* $k > 0$ *because the marginal return to capital approaches infinity at* 0.

Suppose at the optimal solution $l_k > 0$. *Consider a perturbation as follows: reduce both* m *and* l_k *by* $\varepsilon > 0$ *and increase* l_ρ *by* ε*; keep* k, b *constant. This is feasible because both* m *and* l_k *are strictly positive. One can verify that all constraints are satisfied. This perturbation increases profit by* $(R-1)\varepsilon > 0$. *Contradiction! So* $l_k = 0$.

If at the optimal solution $l_\rho < \xi k$, *then a perturbation of increasing* l_ρ *by* $\varepsilon > 0$ *and decreasing* m *by* ε *increases profit of the firm.*

Proposition 1 *Under Assumption 1,*

1. *A tightening of the credit limit reduces capital investment:*

$$\frac{\partial k}{\partial \xi} > 0.$$

2. *A tightening of the credit limit increases bond issuance:*

$$\frac{\partial b}{\partial \xi} < 0.$$

Proof 2 *From Lemma 3, we know that* $l_\rho = \xi k$ *and* $m = (\rho - \xi)k$. *Plug them into the objective function of the firm and take the first order condition, we get:*

$$\alpha A k^{\alpha-1} = R(\rho - \xi + 1) + \xi = R(\rho + 1) - (R-1)\xi$$

So the marginal cost of capital increases as ξ *decreases. Hence* k *decreases as* ξ *decreases.*

Because $b = R(\rho - \xi + 1)k = R(\rho - \xi + 1)[\frac{R(\rho+1)-(R-1)\xi}{\alpha A}]^{1/(\alpha-1)}$

The intuition is as follows. As credit limit tightens, firms are forced to hold more cash per unit of capital investment. This increases the opportunity cost of investment and reduces optimal capital stock.

2.4 General Model

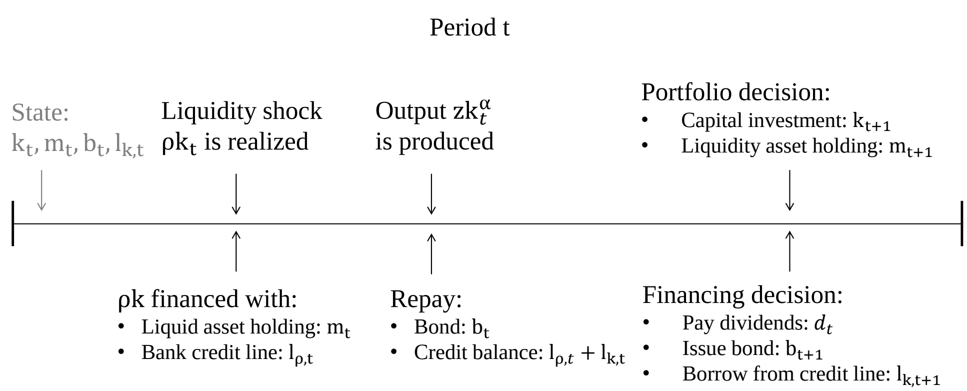
In this section, we extend the two-period example into an infinite-horizon economy. In the two-period example, liquidity shock ρ is assumed to be a constant. In this section, we allow liquidity shocks to be stochastic (ρ is a random variable). This generalization is consistent with the corporate liquidity management literature. It also facilitates our calibration in Section 2.5.

2.4.1 Firms

There is a unit measure of firms in the economy. Firms operate a decreasing returns to scale production technology $y = zk^{\alpha}$ converting capital k into output y. z is an economy-wide productivity level (which is assumed be to a constant). All the firms are producing a homogeneous output.

The timing of the model is in Figure 2.6.

Figure 2.6: Timing



Firms walk into period t with capital stock k_t, cash holdings m_t, corporate bond outstanding b_t, and credit line debt outstanding l_t^k. A liquidity shock ρk_t is realized, where ρ is i.i.d. across firms and over time with distribution $g(\rho) \sim N(\mu_\rho, \sigma_\rho^2)$. The liquidity shock is the amount of money that a firm has to pay right away before the project can be continued. It can be financed by either drawing down the remaining credit line $\left(\xi k_t - l_t^k\right)$ (we will explain in the next subsection 2.4.2 the interpretations for this credit limit ξk_t), or using cash holdings m_t. We assume that bond issuance is less flexible and hence cannot be contingent on the realization of the liquidity shock. This is motivated by the lengthy Security and Exchange Commission filling procedure and the high flotation cost such as underwriting fees, legal fees and registration fees associated with bond issuance, as documented in Krishnaswami et al. (1999).

If a firm manages to finance the liquidity shocks, it proceeds to the production stage and produces $f(k_t)$. It pays back banks and bondholders, issues new bonds b_{t+1}, draws down next-period credit line l_{t+1}^k, pays out dividends d_t, and makes capital investment k_{t+1} and cash holdings m_{t+1} decisions for the next period. If a liquidity shock is not financed, the firm will lose all its capital stock k_t and cash holdings m_t. The firm exits the market and is replaced by a new firm.

In order to get a stationary distribution of the net worth of the firms, we assume that there is some exogenous bankruptcy probability θ for each firm in each period. Firm's optimization problem could be summarized as the following Bellman equation:

$$V(k,m,b,l_k) = \max_{\rho^*,d(\rho),b'(\rho),l_k'(\rho),k'(\rho),m'(\rho)} \int_{\rho\le\rho^*} \left[d(\rho)+\beta\theta V\left(k'(\rho),m'(\rho),b'(\rho),l_k'(\rho)\right)\right] g(\rho)d\rho$$

$$\text{s.t.} \quad \rho^* k \le m + (\xi k - l_k) \tag{2.4}$$

$$d(\rho)+k'(\rho)+m'(\rho)+\rho k+b+l_k \tag{2.5}$$

$$\le q_b\cdot b'(\rho)+q_{l_k}\cdot l_k'(\rho)+f(k)+(1-\delta)k+m, \forall\rho\le\rho^* \tag{2.6}$$

$$q_b = qG\left(\frac{m'(\rho)+\xi k'(\rho)-l_k'(\rho)}{k'(\rho)}\right) \tag{2.7}$$

$$q_{l_k} = qG\left(\frac{m'(\rho)+\xi k'(\rho)-l_k'(\rho)}{k'(\rho)}\right) \tag{2.8}$$

where ρ^* is the cutoff level for the liquidity shocks chosen by the firm: a liquidity shock of ρk will be financed if and only if $\rho\le\rho^*$. $d(\rho)$ is the dividend distributed to the households, $k'(\rho), m'(\rho)$ are capital investment and cash holdings for the next period, $b'(\rho)$ is the issuance of new bonds, and $l_k'(\rho)$ is the draw down of the new credit line. Equation (2.4) is the credit limit on bank credit lines and Equation (2.5) is the budget constraint of the firm. Equation (2.7) and Equation (2.8) are interest rate schedules on bonds and credit lines, which are the risk-free rate q adjusted for the probability of going bankruptcy of the firm. Note that the portion of the credit line l_k used for investment purpose is an inter-period loan and carries an interest rate of $\frac{1}{q_{l_k}}$, while the portion l_ρ used for financing the liquidity shock is an intra-period loan and carries an interest rate of 1.

To simplify the optimization problem of the firm, we need the following lemma first:

Lemma 4 $V(k,m,b,l_k) = V(k, m-\varepsilon, b, l_k-\varepsilon), \forall 0<\varepsilon<\min(m,l_k)$, *i.e., value function V only depends on the difference between m and l_k.*

Proof 3 *From the Bellman equation, m and l_k show up only in the credit limit constraint* (2.4) *and the budget constraint* (2.5)*. In both locations, only $m-l_k$ matters.*

This lemma leads to the following proposition stating that bank credit lines will only be used for financing liquidity shocks when cash holdings are positive.

Proposition 2 *If $m'(\rho)>0$, then $l_k'(\rho)=0$*[7] *, i.e. whenever the firm chooses to hold positive cash $m'(\rho)$ for the next period, it won't draw down its credit line $l_k'(\rho)$ for the purpose of capital investment. I.e., it will save all its credit line for financing the liquidity shock in the coming period.*

Proof 4 *Suppose $m'(\rho)>0$ and $l_k'(\rho)>0$. Then the following perturbation will make the firm better off:*

Reduce both $m'(\rho)$ and $l_k'(\rho)$ by $\varepsilon>0$.

This perturbation doesn't affect the continuation payoff to the firm which is proved in Lemma 2. It doesn't affect the price schedule of bond and credit line either which we could see from (2.7) *and* (2.8)*.*

However, it relaxes the budget constraint of this period since $l_k'(\rho)$ carries an interest rate of $\frac{1}{q_{l_k}}>1$. Therefore, firms are strictly better off than before. Contradiction.

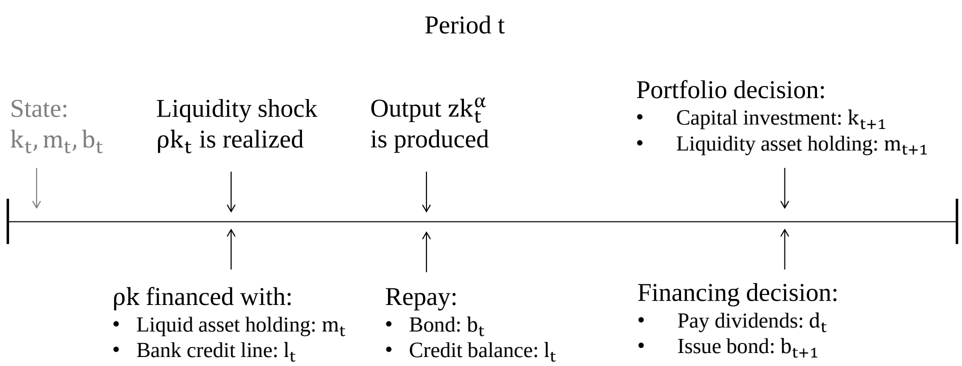
Intuitively, credit line is a cheaper instrument to finance liquidity shocks than cash. This is because credit line (used for financing the liquidity shock) is an intra-period loan carrying an interest rate of 1 while cash has to be set aside and carried over from last period and hence carries an inter-period interest rate. A firm will only hold positive cash if its credit line isn't enough to finance the liquidity shock in the intermediate period. In that case, it will optimally not waste any of the credit line for financing capital investment.

Since our model is trying to explain the cash holdings behavior of firms, parameters are calibrated so that firms have positive cash holdings. As a result, we may assume

[7] *Note: m' and l_k' here denote the policy function for the* next period *cash holding and credit line usage. They are NOT derivatives.*

without loss of generality here that cash holdings are positive and none of the credit line is used for capital investment. This simplifies the timing (Figure 2.7) and firm's Bellman equation to:

Figure 2.7: Timing (simplified)



Firm's Bellman equation:

$$\begin{aligned}
V(k,m,b) \quad &= \quad \max_{\rho^*,d(\rho),b'(\rho),k'(\rho),m'(\rho)} \int_{\rho\leq\rho^*} \left[d(\rho) + \beta\theta V\left(k'(\rho), m'(\rho), b'(\rho)\right)\right] g(\rho)d\rho \\
\text{s.t.} \quad & \quad \rho^* k \leq m + \xi k \\
& \quad d(\rho) + k'(\rho) + m'(\rho) + \rho k + b \\
& \quad \leq q_b \cdot b'(\rho) + f(k) + (1-\delta)k + m, \forall \rho \leq \rho^* \\
& \quad q_b = qG\left(\frac{m'(\rho) + \xi k'(\rho)}{k'(\rho)}\right)
\end{aligned}$$

2.4.2 Bonds and Bank Credit Lines

As discussed above in firm's problem, firms issue one-period corporate bonds $b'(\rho)$. Interest schedule on bonds takes into account the bankruptcy probability of the firms:

$$q_b = qG\left(\frac{m'(\rho) + \xi k'(\rho)}{k'(\rho)}\right)$$

Bank loans come in the form of credit lines with credit limits. For a firm with capital stock k_t, its credit limit is ξk_t. We motivate this credit limit as a result of renegotiation process between firms and banks when credit line debt becomes due. To be concrete, we assume that firms cannot commit to pay back their credit lines. When the credit line debt becomes due, firms have the option of renegotiating with banks about how much money they will pay back. If the renegotiation fails, banks will seize firm's capital stock k_t and liquidate it in the market at some exogenous price ξ. Therefore, no matter how much banks lend to the firms during the lending stage, ξk_t is the maximum amount of money banks can get back during the pay-back stage (outside option of the banks). For simplicity, we assume firms make take-it-or-leave-it offers during the renegotiation stage and hence banks will get no more than their outside option. This motivates the credit limit of ξk_t on the credit lines.

2.4.3 Household

Household's problem is relatively simple. They consume the dividends paid out by the firms and save in the bond market. Households are risk averse and discount future at rate β. In the stationary equilibrium risk free rate $q = \beta$.

$$\begin{aligned} W(b) &= \max_{c,b'} U(c) + \beta W(b') \\ \text{s.t.} &\quad c + b \leq qb' + D \end{aligned}$$

where D is the aggregate dividends paid out to the households. Denote the policy functions of the households by c^h, b^h.

2.4.4 Equilibrium

Definition 2 *A stationary equilibrium is a collection of functions $V, W, m', d', k', b', c^h, b^h$ and the risk free rate q such that:*

1. *Given q, V, m', d', k', b' solve individual firm's problem;*
2. *Given q, W, c^h, b^h solve household's problem;*
3. *Goods market and bonds market clear.*

2.5 Quantitative Analysis

To facilitate a comparison of our credit line shocks with shocks to the standard borrowing constraint (henceforth standard credit shocks), we add a standard borrowing constraint

(2.9) to the previous analyzed problem. Specifically, we numerically solve the following problem:

$$\begin{aligned}
V(k,m,b) \quad &= \quad \max_{d(\rho),b'(\rho),k'(\rho),m'(\rho)} \int_{\rho k \leq m+\xi k} \left[d(\rho) + \beta\theta V\left(k'(\rho), m'(\rho), b'(\rho)\right)\right] g(\rho) d\rho \\
\text{s.t.} \quad & \quad d(\rho) + k'(\rho) + m'(\rho) + (\rho k - m) + b \leq q_b \cdot b'(\rho) + f(k) + (1-\delta)k \\
& \quad q_b = qG\left(\frac{m'(\rho) + \xi k'(\rho)}{k'(\rho)}\right) \\
& \quad b \leq \eta k
\end{aligned} \tag{2.9}$$

We simulate a panel of 100,000 firms to the stationary distribution. Then we study the impulse responses of this economy to various shocks.

2.5.1 Calibration

We choose the following parameters independently of the model equilibrium. Discount factor β is set to 0.96 to match annual interest rate 4%. Depreciation rate is set to 5%, as estimated by Nadiri and Prucha (1996). Curvature of the production function α is set to 0.7, as in Hennessy and Whited (2006) and Cooper and Ejarque (2003).[8] Productivity z is normalized to 1. Exogenous bankruptcy rate is set to a small positive number0.01 to ensure the existence of a stationary equilibrium.

Next we move on to the set of parameters that we estimate using firm-level data from COMPUSTAT. The most important task is to estimate the structure of corporate liquidity shocks. To our knowledge, this has not been done in the literature. Current liabilities are what a firm needs to pay within a relatively short period of time. Hence we use the ratio of current liabilities over total asset as proxy for liquidity shocks of the period. We estimate the mean and variance of liquidity shocks using cross-sectional mean and variance of firm's current liabilities over total asset ratio, respectively. The assumption behind our estimation is that all firms are subject to the same process governing the idiosyncratic liquidity shock.

In our model there are two types of borrowing constraints: standard borrowing constraint and credit line limit. We estimate the former using the leverage ratio and the latter using credit line over capital ratio as documented in Sufi (2009). A summary of our calibration is presented in Table 2.1.

[8] For a detailed discussion of choice of α in production functions without labor, see Covas and Haan (2012).

Before explaining the results of our quantitative exercise, it is useful to discuss how well the model does in capturing some of the key moments in data. One way of checking the validity of our estimates is to compare the cash over capital ratio of our model to the data. In our calibrated model, cash over capital ratio is 0.32, while in the data it is 0.31.

Table 2.1: Calibration

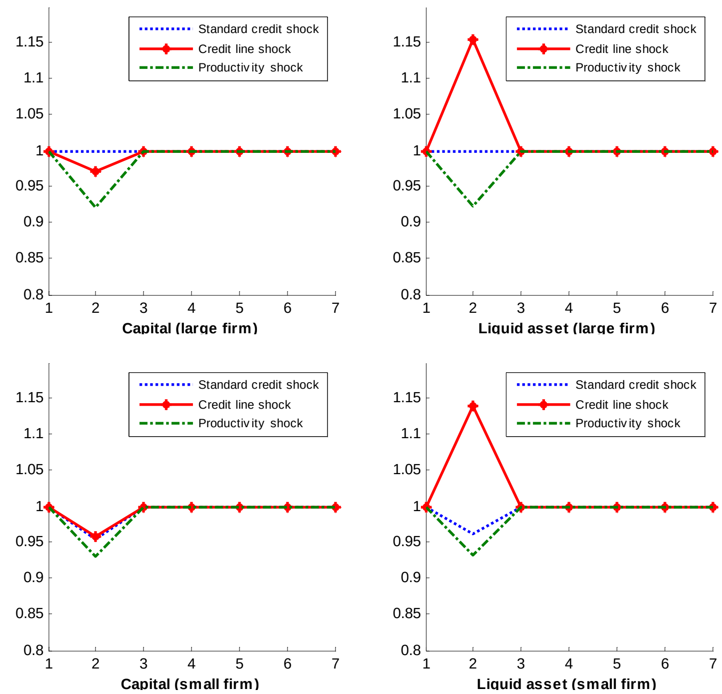
Calibrated parameters			Source		
Discount factor	β	0.96	Annual interest rate 4%		
Depreciation rate	δ	0.05	Nadiri and Prucha (1996)		
Technology	α	0.7	Cooper and Ejarque (2003)		
Productivity	z	1	Normalization		
Exogenous death rate	θ	0.01	Guarantee existence of stationary equilibrium		
Estimated parameters			**Target moments**	**Model**	**Data**
Mean of liquidity shocks	μ_c	0.35	Cross-sectional mean of current liabilities/asset ratio	0.26	0.23
Variance of liquidity shocks	σ_c	0.032	Cross-sectional variance of current liabilities/asset ratio	0.018	0.02
Credit line constraint	ξ	0.57	Mean of available credit line/capital ratio	0.56	0.54
Standard borrowing constraint	η	1.1	Mean of leverage ratio	0.73	0.61
			Untargeted moment		
			Mean of cash/capital ratio	0.32	0.31

2.5.2 Impulse Response

Key Result

Our key results are illustrated in Figure 2.8. The top row is for large firms (net worth at top 25% of firms). Standard credit shocks (tightening of bond borrowing constraint) are unable to generate a reduction in capital expenditure, or an increase in liquid assets holdings for large firms, whereas credit line shocks are able to generate both.

Figure 2.8: Capital Investment and Liquid Asset Holding under Different Shocks



The intuition is as follows. Large firms have accumulated enough net worth so that their borrowing constraints are no longer binding. Therefore, standard credit shocks have no effects on their investment decision. However, credit line shocks do have an effect on large firms by forcing them to hold more cash. Costly cash holdings increase the effective marginal cost on capital investment. Therefore, firms optimally choose to reduce investment.

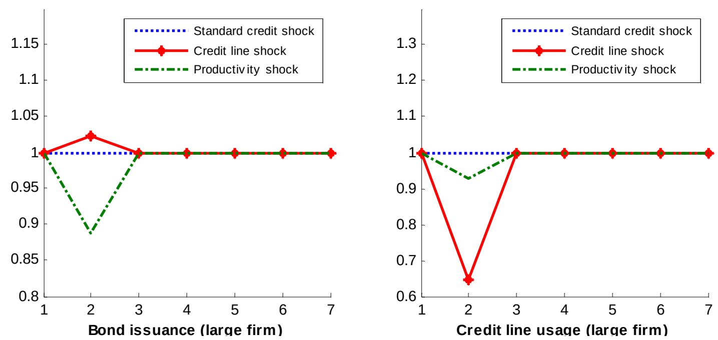
For small firms (net worth at bottom 25% of firms), both standard credit shocks and credit line shocks reduce their capital investment because these firms are both borrowing constrained and liquidity constrained. But their implications on corporate cash holdings are different. Standard credit shocks reduce corporate cash holdings because they reduce corporate investment, whereas credit line shocks force firms to hold more cash per unit of capital.

Finally, productivity shocks lead to a reduction in capital for both large and small firms, consistent with the data. However, productivity shocks also imply a counterfactual reduction in liquid assets holdings.

Prediction of Loan-Bond Composition

Adrian, Colla and Shin (2013) documents that loan financing collapses whereas bond financing remains strong. Our credit line shocks match that feature in the data, whereas standard credit shocks and productivity shocks do not (Figure 2.9). As credit line shocks hit, firms' demand for liquid assets rises. This increases the demand for corporate bonds despite a reduction in corporate investment.

Figure 2.9: Corporate Bond and Bank Loan under Different Shocks



2.6 Conclusion

In this paper, we document a significant reduction in capital investment, an increase in liquid assets holdings, and an increase in relative amount of corporate bonds issuance to bank loans issuance by self-financing firms during the Great Recession.

These three facts cannot be simultaneously accounted for in models with collateral constraints or productivity shocks. To explain the three facts, we propose a model where firms' capital stock is subject to *liquidity shocks.* Bank credit lines and liquid assets are substitutes for insuring against liquidity shocks and safeguarding capital investment. Given the amount of credit lines available, firms have to bundle one unit of capital with certain units of liquid assets to cover the remaining liquidity risk. When banks tighten the credit lines, firms have to bundle extra units of liquid assets with each unit of capital to cover the increased liquidity risk. Holding liquid assets, however, is costly because of their low return. This increases the marginal cost of capital investment. Hence, firms optimally reduce their capital investment.

Then we calibrate our model to firm-level balance sheet information from COMPUSTAT, and compare the predictions of negative productivity shocks in a standard RBC model, tightening of the collateral constraints in a standard model of financial frictions, and credit line shocks in our model. We find that standard credit shocks do not affect self-financing firms, leaving their capital expenditure and cash holdings unchanged; productivity shocks generate a decrease of capital expenditure across firm sizes. But they imply a counter-factual reduction in firms' liquid assets holdings; credit line shocks generate a decrease in capital expenditure as well as an increase in liquid assets holdings across firm sizes. Credit line shocks also generate a compositional change between loans and bonds as in the data.

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Appendix A

Appendix

A.1 Employment Recovery Across Recessions

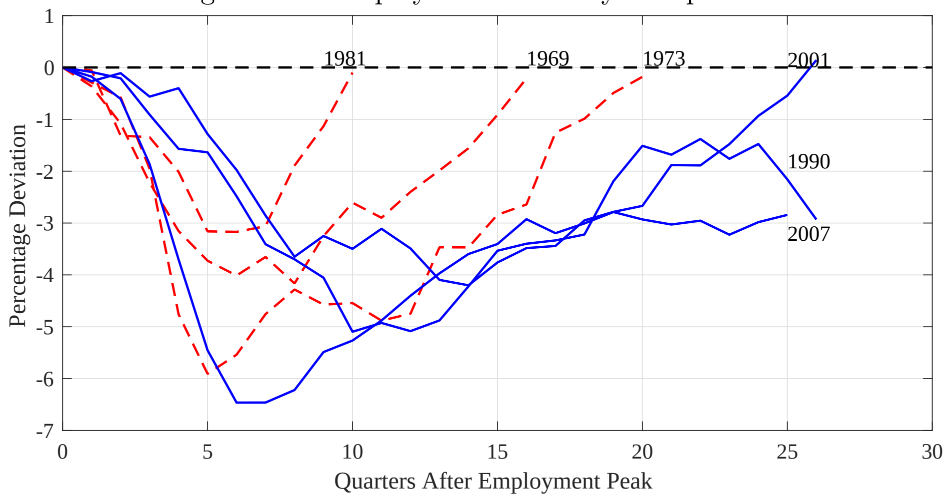
In this section, additional comparison of the speed of employment recovery is provided. In Section 1.3, employment is measured as total hours of all persons in the non-farm business sector. Total hours worked is detrended using an HP filter. In Figure 1.1 of that section, we compare the recovery paths of total hours for recessions before and after the mid-1980s. For clarity, only the average recovery paths for recessions before and after the mid-1980s are displayed. In Figure A.1, we include the recovery paths for all recessions. If a subsequent recession occurs before employment has fully recovered, the recovery path is truncated.

Besides total hours, another commonly used measure for employment is total employment level in the non-farm business sector. Employment exhibits a similar pattern to total hours. For the recessions prior to the mid-1980s, employment recovers to pre-recession peaks within four years after the NBER recession end date. In contrast, for the three recessions after the mid-1980s, employment is significantly below the pre-recession peak even six years after NBER recession end date.

A.2 Average Labor Productivity and GDP

In this section, time series plots of HP filtered average labor productivity and GDP are compared for the pre-1985 and the post-1985 periods. Labor productivity is calculated

Figure A.1: Employment Recovery Comparison



Source: Bureau of Labor Statistics, Authors' calculations

as real output per hour of all persons from the non-farm business sector. We apply an HP filter and plot the residual.

Prior to the mid-1980s, the pattern of labor productivity tracks that of GDP closely during business cycles. Labor productivity falls during recessions and recovers at approximately the same speed as GDP. This results in a procyclical pattern of labor productivity.

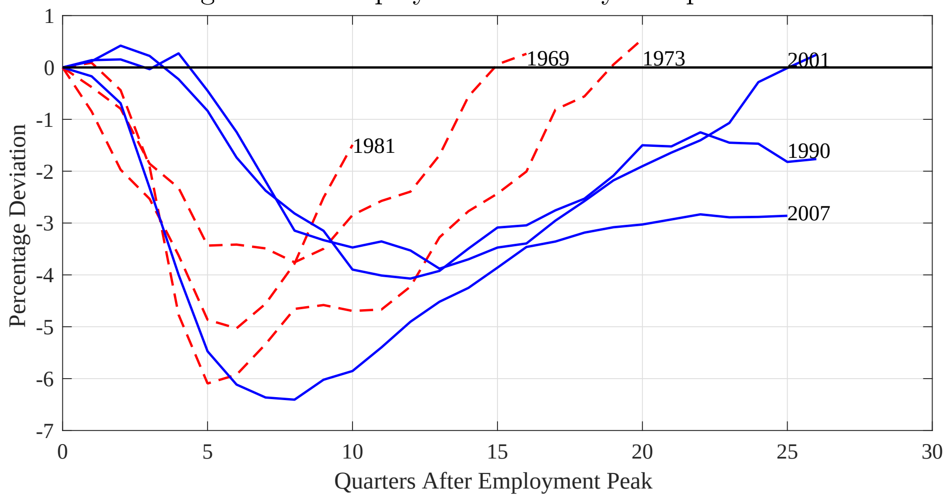
After the mid-1980s, the correlation between GDP and labor productivity becomes weaker. In a recession, labor productivity drops with GDP. However, during the recovery, labor productivity recovers much faster than GDP, resulting in no correlation or even a negative correlation between the two series.

A.3 Proof of Lemma 1

Proof 5 *The break-even condition of financial intermediaries is:*

$$w^S n^S + w^J n^J = (1 - F(z^*)) \left(w^S n^S + w^J n^J \right) R + \int^{z^*} z \left[n^S \theta^S + n^J \theta^J \right]^\alpha f(z) dz$$

Figure A.2: Employment Recovery Comparison



Source: Bureau of Labor Statistics, Authors' calculations

The right hand side of this condition can be simplified into:

$$\begin{aligned}
&(1-F(z^*))\left(w^S n^S + w^J n^J\right) R + \int^{z^*} z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)dz \\
&= \int \min\left\{z\left[n^S\theta^S + n^J\theta^J\right]^\alpha, \left(w^S n^S + w^J n^J\right) R\right\} f(z)dz \\
&= \int z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)\,dz - \int \max\left\{0, z\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\right\} f(z)dz.
\end{aligned}$$

From this condition, we obtain:

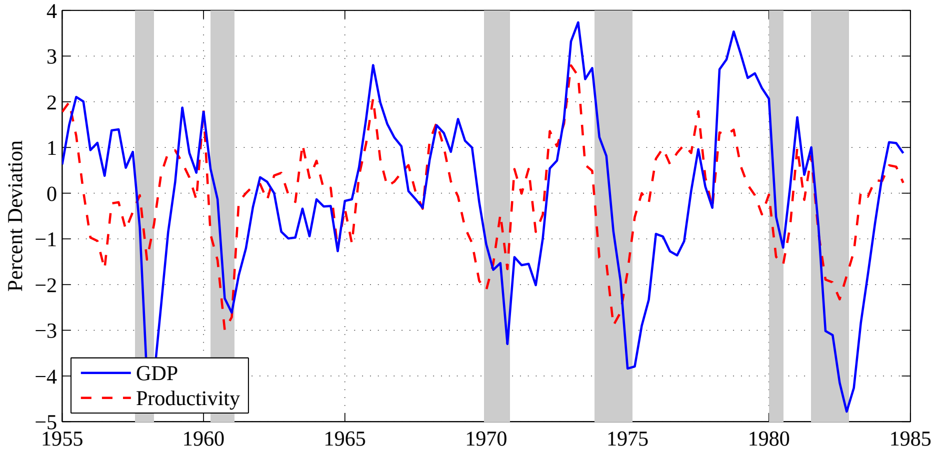
$$\begin{aligned}
&\int \max\left\{0, z\left[n^S\theta^S + n^J\theta^J\right]^\alpha - R\left(w^S n^S + w^J n^J\right)\right\} f(z)dz \\
&= \int z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)\,dz - \left(w^S n^S + w^J n^J\right).
\end{aligned}$$

Plugging this into the Bellman's equation of firms, we get:

$$\begin{aligned}
& V^1\left(n^S\right) \\
= \;& \max_{n^J}\left\{\int_{z^*}\left[z\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\right] f(z)dz\right. \\
& \left. +\beta\left[1-(1-F(z^*))(1-\xi)\right] V^1(0) + \beta(1-F(z^*))(1-\xi) V^1\left(n^{S'}\right)\right\} \\
= \;& \max_{n^J}\left\{\int z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)\,dz - \left(w^S n^S + w^J n^J\right)\right. \\
& \left. +\beta\left[1-(1-F(z^*))(1-\xi)\right] V^1(0) + \beta(1-F(z^*))(1-\xi) V^1\left(n^{S'}\right)\right\}
\end{aligned}$$

as desired.

Figure A.3: Before 1985: Average Labor Productivity and GDP (HP-Filtered)



Source: Current Population Survey and NBER recession dates. Author's calculations

A.4 Proof of Lemma 2

Proof 6 *Assuming there is no risk of default or exogenous separation* $\xi = 0$*, constant returns to scale* $\alpha = 1$*, and that all junior workers transition to senior workers in one period* $\zeta = 1$*, we can analytically solve for the value function and wage rates using a guess-and-verify approach:*

$$\begin{aligned} V^1\left(n^S\right) &= E\left(z\right)\left(1-\phi\right)\left(\theta^S-\theta^J\right)n^S \\ w^S\left(n^S,n^J\right) &= E\left(z\right)\left\{\theta^S-\left[1-\beta\left(1-\delta\right)\right]\left(1-\phi\right)\left(\theta^S-\theta^J\right)\right\} \\ w^J\left(n^S,n^J\right) &= E\left(z\right)\left[\theta^J+\beta\left(1-\delta\right)\left(1-\phi\right)\left(\theta^S-\theta^J\right)\right] \end{aligned}$$

which implies:

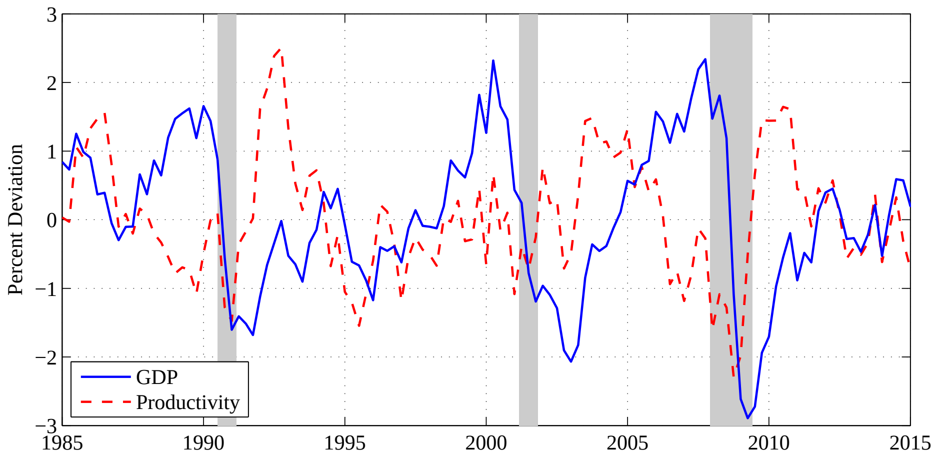
$$\left[w^S-w^J\right]=\phi\left[E\left(z\right)\theta^S-E\left(z\right)\theta^J\right].$$

A.5 Recursive Stationary Equilibrium

Definition 3 *A recursive stationary equilibrium consists of:*

- *Value functions:*
  - $V^1\left(n^S\right)$ *and* $V^2\left(n^S,n^J,w^S,w^J,R,z^*\right)$ *for firms*

Figure A.4: After 1985: Average Labor Productivity and GDP (HP-Filtered)



Source: Current Population Survey and NBER recession dates. Author's calculations

- $S^1\left(n^S\right)$ *and* $S^2\left(n^S, n^J, w^S, w^J, R, z^*\right)$ *for senior workers*
- $J^1\left(n^S\right)$ *and* $J^2\left(n^S, n^J, w^S, w^J, R, z^*\right)$ *for junior workers*

- *Policy function:*
  - $n^J\left(n^S\right)$ *for firms*

- *Wage schedules:*
  - $w^S\left(n^S, n^J\right)$ *for senior workers*
  - $w^J\left(n^S, n^J\right)$ *for junior workers*

- *Interest rate schedule and bankruptcy cutoff:* $R\left(n^S, n^J\right)$ *and* $z^*\left(n^S, n^J\right)$

- *A distribution of firm sizes:* $g\left(n^S\right)$

- *Aggregates: consumption* C*, output* Y*, labor demand* L^d*, and labor supply* L^s

such that:

- *Given the schedules of wage rates, interest rate and bankruptcy cutoff, value function* $V^1\left(n^S\right)$ *and policy function* $n^J\left(n^S\right)$ *solve firm's optimal hiring problem*

- *The schedules of wage rates, interest rates and bankruptcy cutoffs jointly satisfy wage bargaining and break-even equations* (1.1), (1.2), (1.3), *and* (1.4)

- *The distribution of firms $g\left(n^S\right)$ is consistent with the policy function of firms:*

$$g\left(n^{S'}\right)=\begin{cases}\int_{n^S}\left[1-(1-\xi)\left(1-F\left(z^*\left(n^S,n^J\left(n^S\right)\right)\right)\right)\right]g\left(n^S\right)dn^S & \text{if } n^{S'}=0\\ \int_{n^S}\left\{\begin{array}{c}(1-\xi)\left(1-F\left(z^*\left(n^S,n^J\left(n^S\right)\right)\right)\right)\\ \times\mathbf{1}\left\{n^{S'}=(1-\delta)\left(n^S+\zeta n^J\right)\right\}g\left(n^S\right)dn^S\end{array}\right\} & \text{if } n^{S'}>0\end{cases}$$

- *Aggregate output is consistent with the distribution of firms:*

$$Y=\int_{n^S}E(z)\left[\theta^S n^S+\theta^J n^J\left(n^S\right)\right]^\alpha g\left(n^S\right)dn^S$$

- *Labor demand is consistent with the distribution of firms:*

$$L^d=\int_{n^S}\left[n^S+n^J\left(n^S\right)\right]g\left(n^S\right)dn^S$$

- *Labor supply is consistent with household's intra-period Euler equation:*

$$u_c\left(C,L^s\right)\bar{w}=u_L\left(C,L^s\right)$$

where $\bar{w}$ is the average wage rate in the economy defined by

$$\bar{w}\equiv\frac{\int_{n^S}\left[w^S\left(n^S,n^J\left(n^S\right)\right)n^S+w^J\left(n^S,n^J\left(n^S\right)\right)n^J\left(n^S\right)\right]g\left(n^S\right)dn^S}{L^d}$$

- *Labor market clears:*

$$L^d=L^s$$

- *Goods market clears:*

$$Y=C$$

A.6 Determining θ^S/θ^J Using the CPS

As outlined in Section 1.6, we use data from the Displayed Workers, Occupational Mobility and Job Tenure supplements of the CPS to infer the relative productivity of senior to junior workers. This supplemental survey has not been administered every year, and the survey questions have varied somewhat over time. The first time these supplemental questions were asked was in 1973. Unfortunately, questions inquiring about displacement as well as prior and current wages were not asked again until 1984. Since then, these questions have continued to be included bi-annually to present.

Following the method used in Topel (1991), we restrict attention to male respondents between the ages of 20 and 60 whose jobs end exogenously. We then deflate nominal wages by the GNP price deflator for consumption expenditure. For these workers we calculate the average change in log weekly wages for the prior and current jobs.

The percentage drop in wages following an exogenous separation is affected by the business cycle. Specifically, the survey conducted immediately following each recession yields a larger drop in wages than the average drop experienced in surrounding observations. We therefore calculate the drop in wages by both including and excluding these points. For the supplemental survey administered prior to the double-dip recession, we find an average change in log weekly wages of -2.7%. For those surveys administered after the double-dip recession, we find an average change in log weekly wages of -14.7%. When the post-recession data points are excluded, we observe an average change in log weekly wages of -11.5%.

A.7 Determining θ^S/θ^J Using the PSID

Altonji and Shakotko (1987) and Altonji and Williams (2005) use the following specification for estimating returns to seniority:

$$W_{ijt} = \beta_0 t + \beta_1 X_{ijt} + \beta_2 T_{ijt} + \epsilon_{ijt} \tag{A.1}$$

where W_{ijt} is the log earning of person i in job j in period t. X is total labor market experience and T is tenure with the employer. The error term ϵ is potentially dependent on individual specific component μ_i and match specific component ϕ_{ij}:

$$\epsilon_{ijt} = \mu_i + \phi_{ij}$$

These individual specific and match specific components are likely to be correlated with both tenure and earnings: an individual with higher ability or a good match between worker and employer are likely to lead to both longer tenure and higher earnings.

To deal with this endogeneity problem, Altonji and Shakotko (1987) and Altonji and Williams (2005) propose an instrumental variable DT_{ij} for tenure. This instrument is defined to be the deviation of tenure T_{ijt} from the mean $\bar{T}_{ij}$ of the sample observations on job match ij. Effectively, incorporating this instrument is equivalent to demeaning Equation (A.1) from its mean over time. Since individual and match specific components in the error term are assumed to be invariant over time, they will be eliminated after this demeaning procedure. Intuitively, although individual and match specific components are likely to affect the earnings of a worker, they stay constant within a worker-job match. Therefore, if we look within each worker-job match, the increase in earnings over time cannot be attributed to those two components. A similar instrument is also used for year.

Altonji and Williams (2005) extend the methodology of Altonji and Shakotko (1987) to newer PSID waves of 1983 to 1991. We further extend their estimations to 2013. Between 1968 and 1997, PSID interviews were conducted annually. Since then, interviews have been biennial. This change in the frequency of interviews does not affect the validity of the regression, because returns to tenure are estimated from earnings gains within worker-job matches.

Summary statistics are included in Table A.1.

The results of our regression analysis are included in Table A.2. We report the estimated returns to tenure at 3, 5, 10 as well as the mean tenure for workers.

A.8 Simulation Results with an Exogenous Firm Exit Shock

In Section 1.7, recessions are simulated as a one-time, unexpected variance shock that disproportionately affects smaller firms. As a robustness check to this particular version of the shock, we analyze a simpler version in which we exogenously force a fraction of firms to exit with equal probability across all firm sizes. We calibrate the fraction of firms which exit to match the percentage drop in output in the data. Figures A.5, A.6, and A.7 display the results from this version of the shock. The resulting business cycle patterns are virtually unaffected compared to our benchmark model found in Section 1.7.3.

Table A.1: Summary Statistics, PSID

Variable	Panel 1: 1975-1982	Panel 2: 1983-2013
ln(Earnings)	2.827	2.883
	(0.458)	(0.573)
Experience	16.540	16.850
	(10.820)	(9.710)
Tenure	8.795	7.952
	(9.083)	(7.934)
Education	12.811	13.540
	(2.430)	(2.123)
Married Now	0.876	0.815
	(0.330)	(0.389)
Union	0.302	0.161
	(0.473)	(0.367)
Disability Affecting Work	0.063	0.073
	(0.244)	(0.261)
Age	36.115	37.018
	(11.240)	(9.973)
North Central Region	0.344	0.327
North Eastern Region	0.224	0.358
Southern Region	0.271	0.279
Western Region	0.160	0.182
Observations	6882	23125

A.9 First Moment TFP Shock

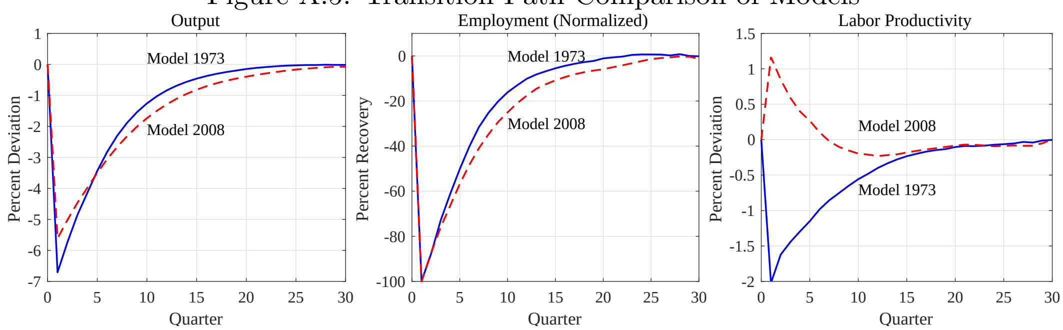
In this section, we demonstrate how a one-period unexpected shock to the mean of TFP would affect our model economy. Since the shock is unexpected, employment is not affected until the first period following the shock. In the period the shock is applied (period 0), the effects of our mechanism are small relative to the shock to TFP. In our benchmark model, we calibrated the magnitude of the shock to match the observed drop in output. If we do the same with the first moment TFP shock, there is insufficient disruption to firms, employment is virtually unaffected, and recovery is immediate. Instead, we calibrate the TFP shock to cause the implied drop in employment for the first period following the shock to match the data.

The results of this shock are displayed in Figure A.8. Matching the drop in employment requires an unreasonably large shock to the mean of TFP causing the observed

Table A.2: IV Estimation of Returns to Tenure

	Panel 1: 1975-1982	Panel 2: 1983-2013
3 Years of Tenure	0.0444	0.0778
	(0.0169)	(0.0106)
5 Years of Tenure	0.0548	0.1143
	(0.0235)	(0.0143)
10 Years of Tenure	0.0422	0.1286
	(0.0353)	(0.0188)
Mean Tenure	0.0484	0.1302
	(0.0324)	(0.0173)

Figure A.5: Transition Path Comparison of Models

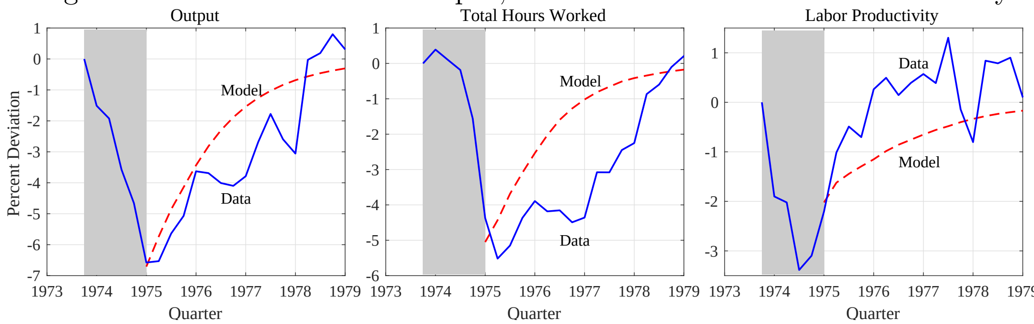


Source: Bureau of Labor Statistics, Authors' calculations

drop in output at the time of the shock to drop much too far relative to the data (left panel). As employment is still at the steady state level this also implies a large drop to average labor productivity (right panel). However, the ensuing recovery is very similar to our benchmark model. Figure A.9 displays the recovery with the initial period removed. When compared with our benchmark model (Figure 1.14), it is observed that these patterns are quite similar.

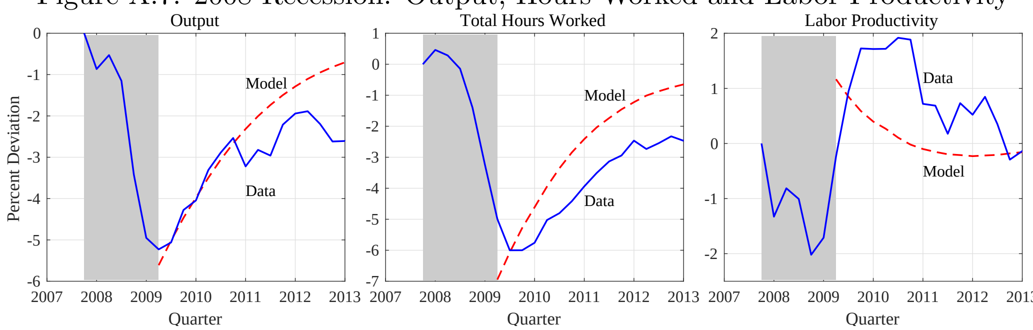
As has been outlined in recent literature, an complete characterization of business cycles includes more than a shock to the first moment of TFP. Bloom et al. (2014) and Stock and Watson (2012) suggest that second moment rather than first moment shocks to TFP may be chiefly responsible for driving business cycle dynamics. Salgado et al. (2016) suggests that skewness in sales growth shifts significantly negative during recessions. Through the lens of our model, both increases in variance and leftward shifts in skewness would cause much larger amounts of exit and distortion to employment than

Figure A.6: 1973 Recession: Output, Hours Worked and Labor Productivity



Source: Bureau of Labor Statistics, Authors' calculations

Figure A.7: 2008 Recession: Output, Hours Worked and Labor Productivity



Source: Bureau of Labor Statistics, Authors' calculations

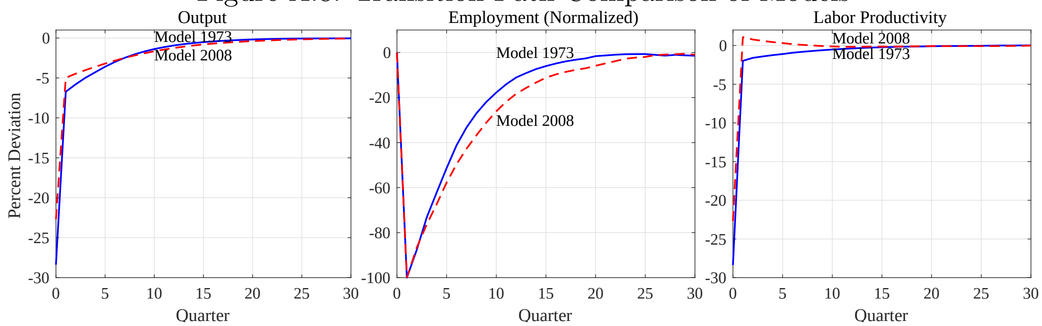
a shift solely to the first moment in TFP. Even still, a one-period unexpected shock to the average of TFP does generate a slower recovery and reduces the cyclicality of labor productivity.

A.10 Simulation Results for Other Recessions

In Section 1.7, we compare the simulation results of our model with the data for the 1973 recession and the 2008 recession. In this section, additional comparisons for the recessions of 1982, 1990, and 2001 are presented.

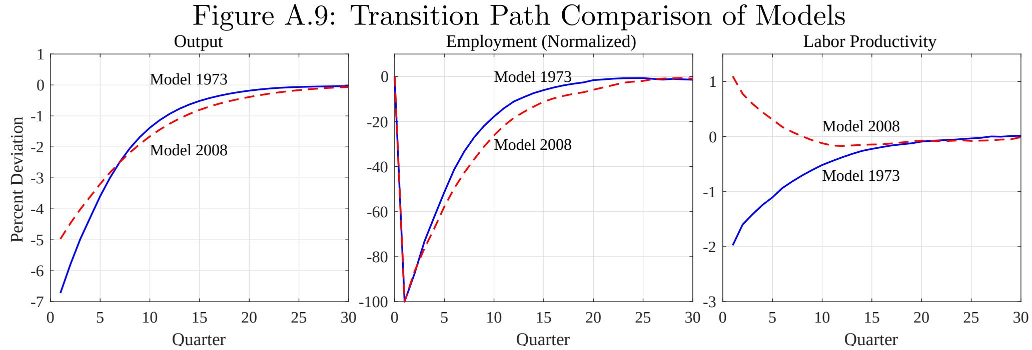
The three recessions prior to the mid-1980s include the 1973 recession, the 1980 recession, and the 1982 recession. The results for the 1973 recession are presented in Section 1.7. The 1980 recession is immediately followed by the 1982 recession so the recovery path is not complete. Below is the comparison of output, unemployment, and

Figure A.8: Transition Path Comparison of Models



Source: Authors' calculations

Figure A.9: Transition Path Comparison of Models



Source: Authors' calculations

labor productivity between data and model for the 1982 recession:

The three recessions after the mid-1980s are the 1990 recession, the 2001 recession, and the 2008 recession. The results for the 2008 recession are presented in Section 1.7. The 1990 and 2001 recessions are plotted below:

A.11 Compare Model and Data Using Employment Level

In Section 1.7, we compare the path of employment recovery between our model and the data. Employment is measured as the percentage deviation of total hours in non-farm business sector from its trend. Another popular measure of employment is the employment level. As a robustness check, we now compare the path of employment recovery in our model with the employment population ratio.

To remove the trend in employment level, we apply an HP filter. Figure A.13 compares the employment recovery between our model and data, using two different

Figure A.10: 1982 Recession

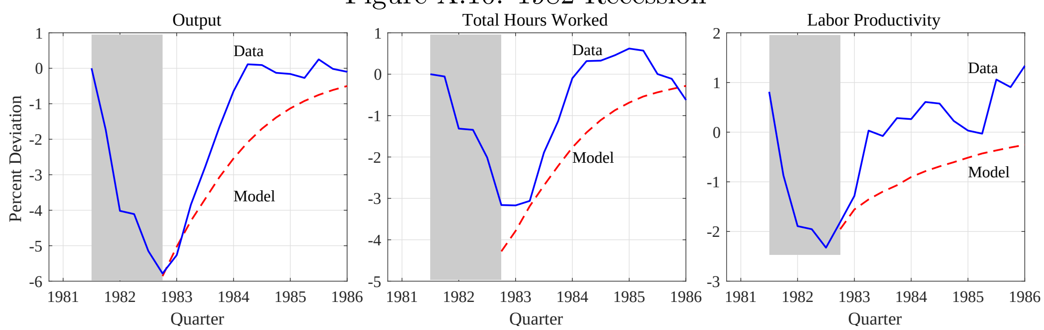
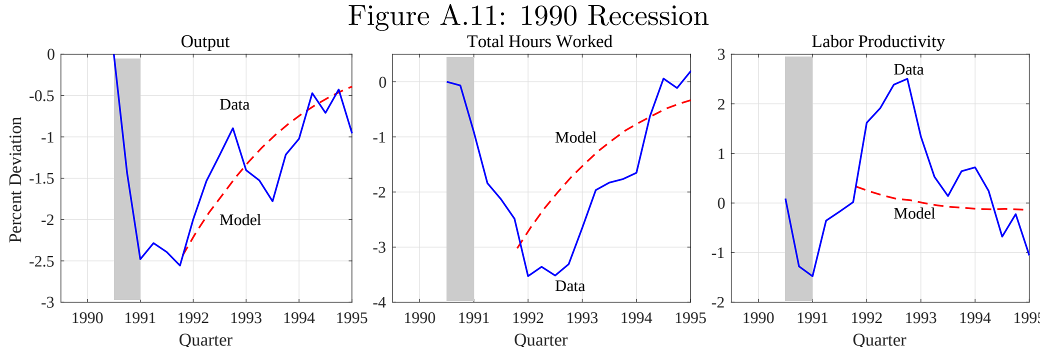


Figure A.11: 1990 Recession



measures of employment. Figure A.14 is the comparison for the 2008 recession.

Figure A.12: 2001 Recession

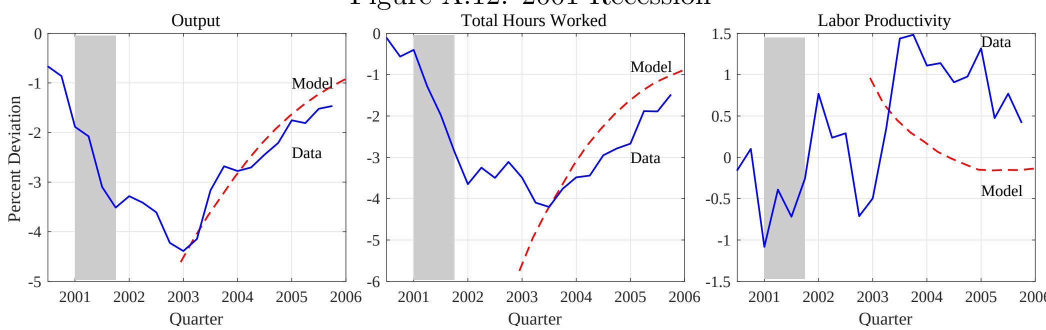


Figure A.13: 1973 Recession

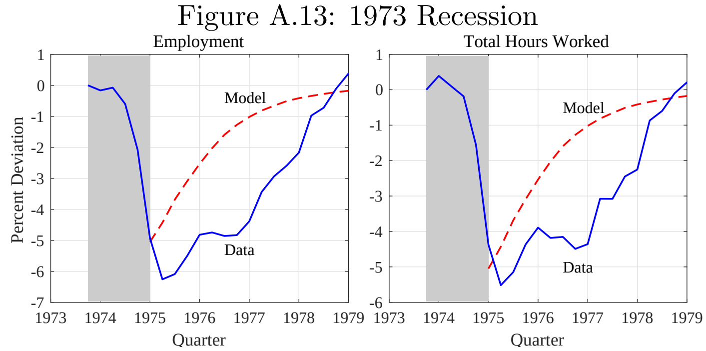


Figure A.14: 2008 Recession